Simplified nonlinear dynamic models for RC structures under blast and impact loading

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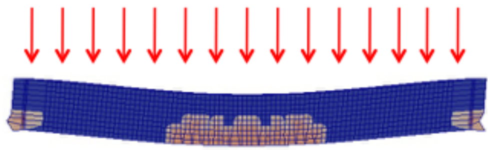
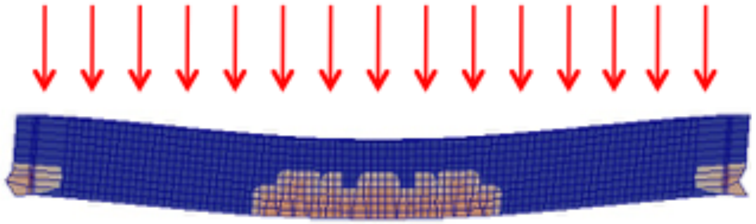


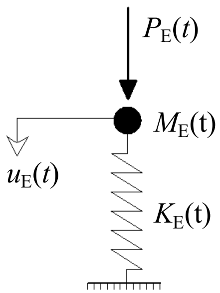
Motivation

Complexity of blast load scenario

Complexity of advanced numerical models



Needs of simplified models

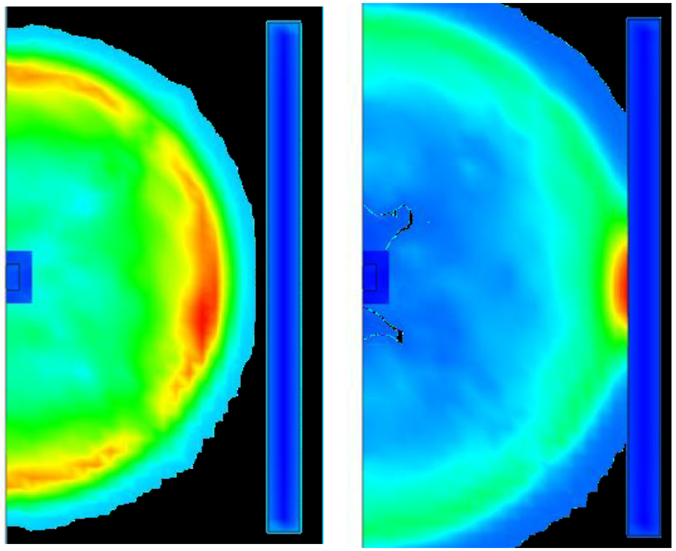


(b)

Outline

- **Main Aim of the Lecture**: *Simplified models for the flexural behaviour of R.C. beams subjected to explosion (and impulsive) verified with experimental results. In all models, account is taken of the effects of strain-rate.*

- Section 1: Dynamic Models
- Section 2: Energy Model
- Section 3: Sensitivity Analysis
- Section 4: Tower Building Case



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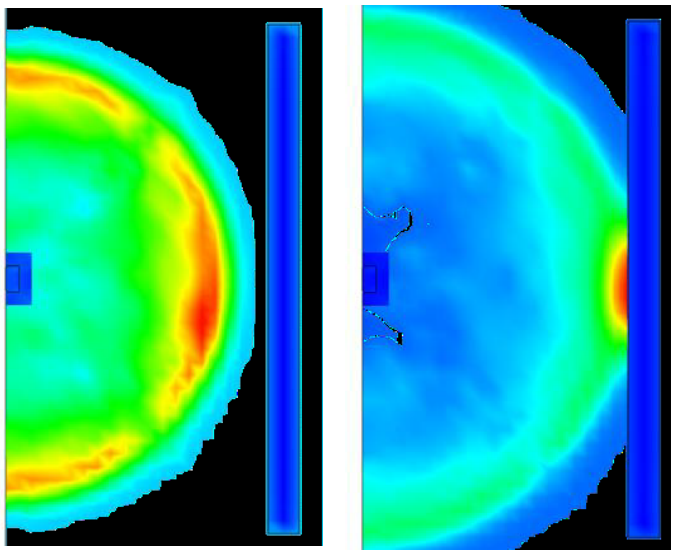
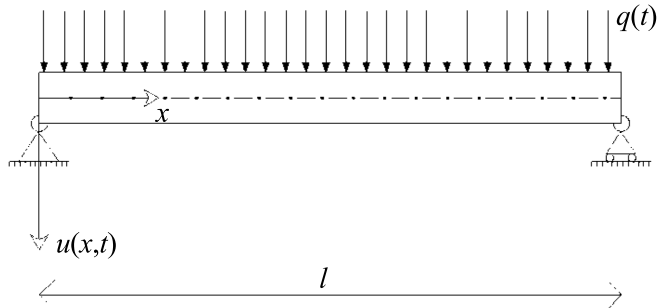


Introduction - Section 1

Section 1: Dynamics Models

- Continuos Beam Model
- SDOF Model
- FEM



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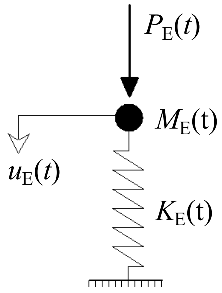

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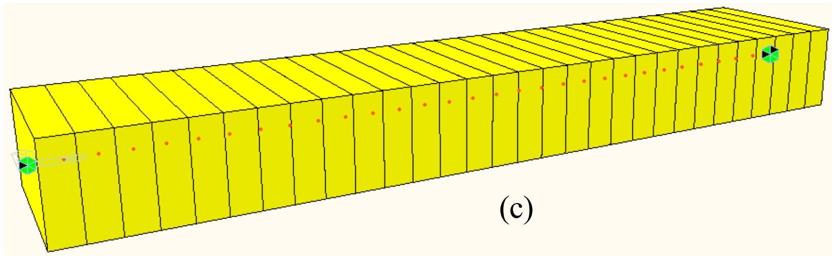
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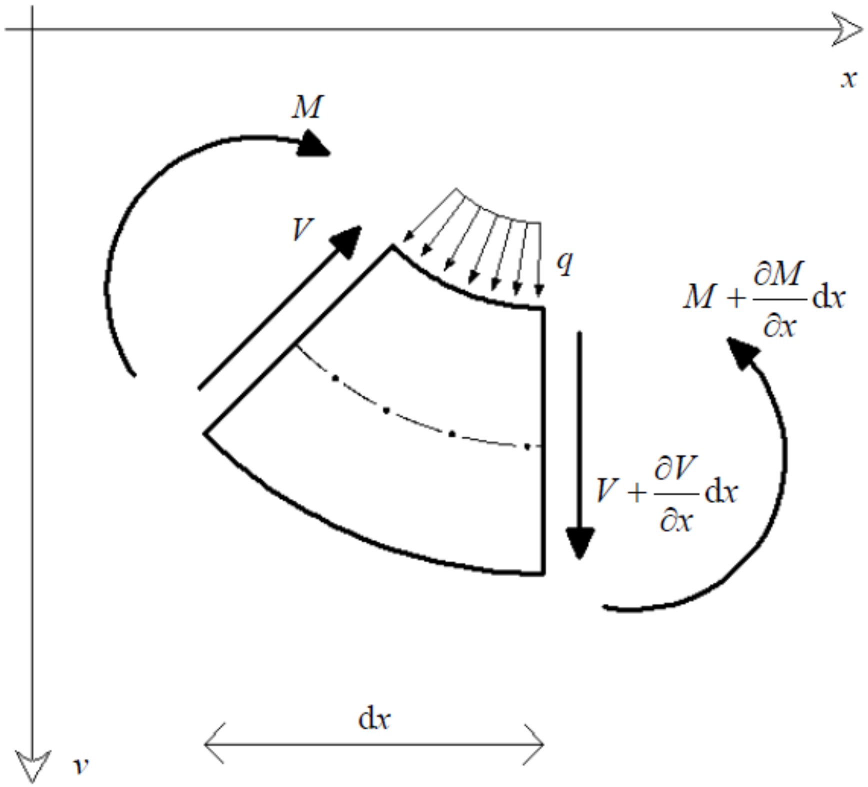
Dynamic model 1: continuous beam

- Euler-Bernulli beam model



$$\frac{\partial^2 M}{\partial x^2} + q = \mu \frac{\partial^2 v}{\partial t^2}$$

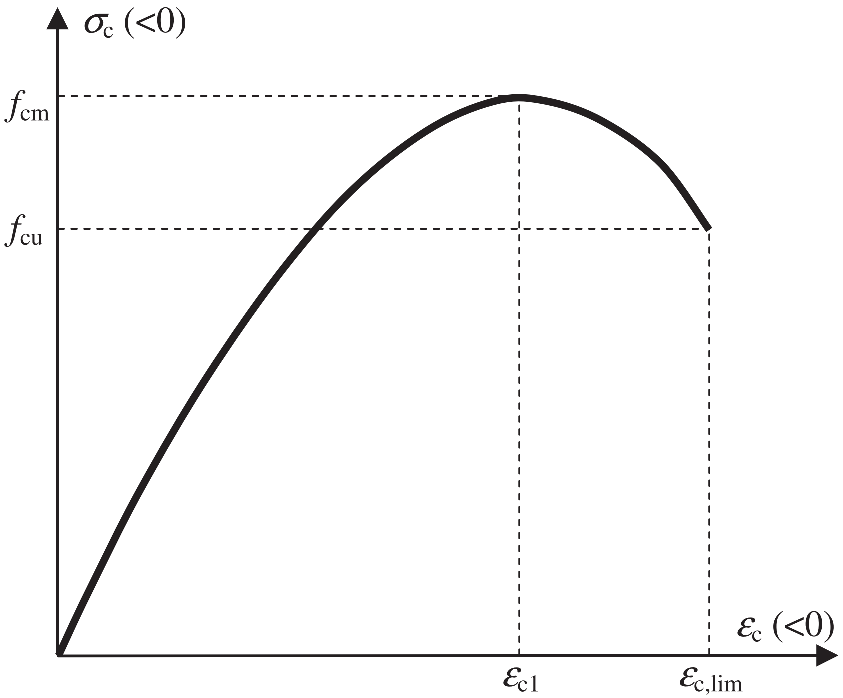
- Plane sections remain plane and perpendicular to the beam axis after deformation

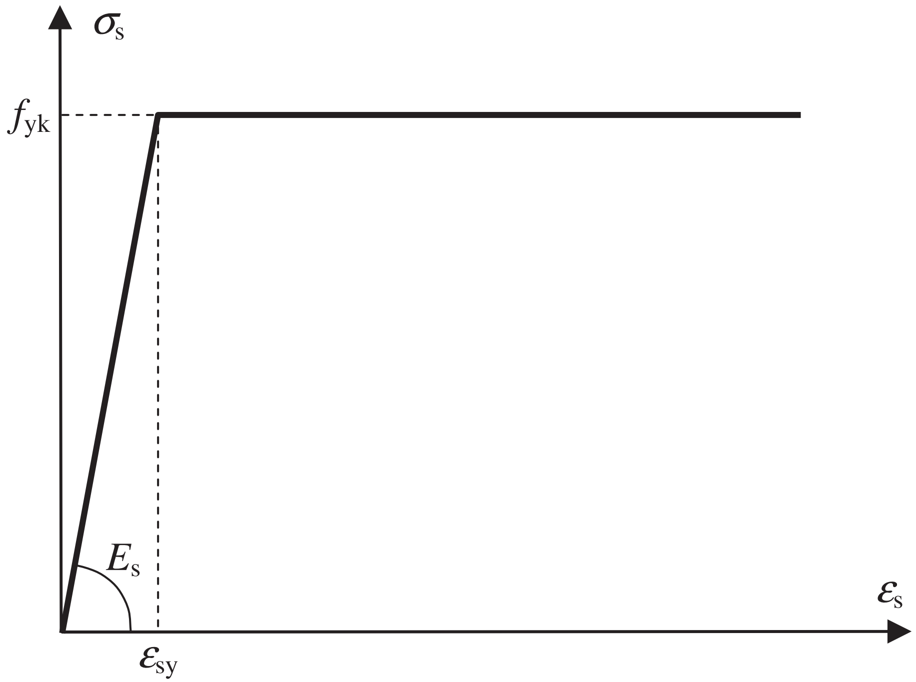
- No shear deformation

Dynamic model 1: continuous beam

Material stress-strain relationships

Model Code. "First complete draft." Bulletin 55 (2010).



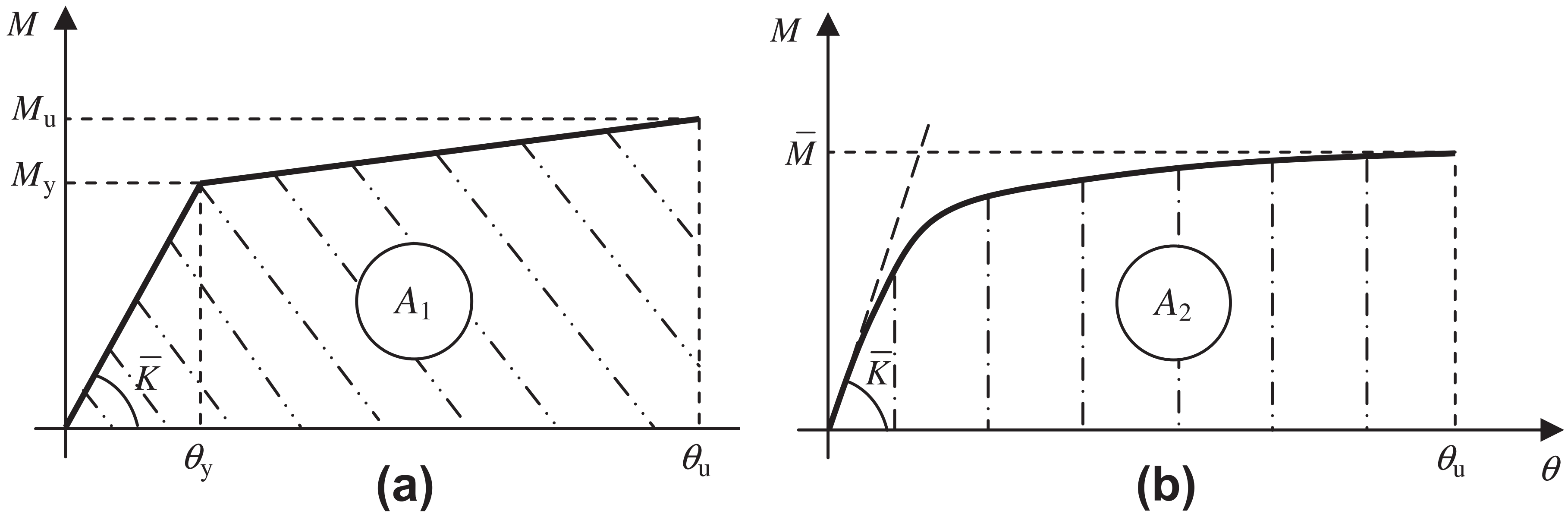


$$\sigma_{\mathrm{c}} = f_{\mathrm{cm}} \frac{k \cdot \varepsilon_{\mathrm{c}} / \varepsilon_{\mathrm{c1}} - \left(\varepsilon_{\mathrm{c}} / \varepsilon_{\mathrm{c1}}\right)^2}{1 + (k-2) \cdot \varepsilon_{\mathrm{c}} / \varepsilon_{\mathrm{c1}}} \quad \text{for } |\varepsilon_{\mathrm{c}}| < |\varepsilon_{\mathrm{c,lim}}|$$

$$f_s = E_s \cdot \varepsilon_s, \quad f_s = f_y$$

Dynamic model 1: continuous beam

- Bending moment- curvature law

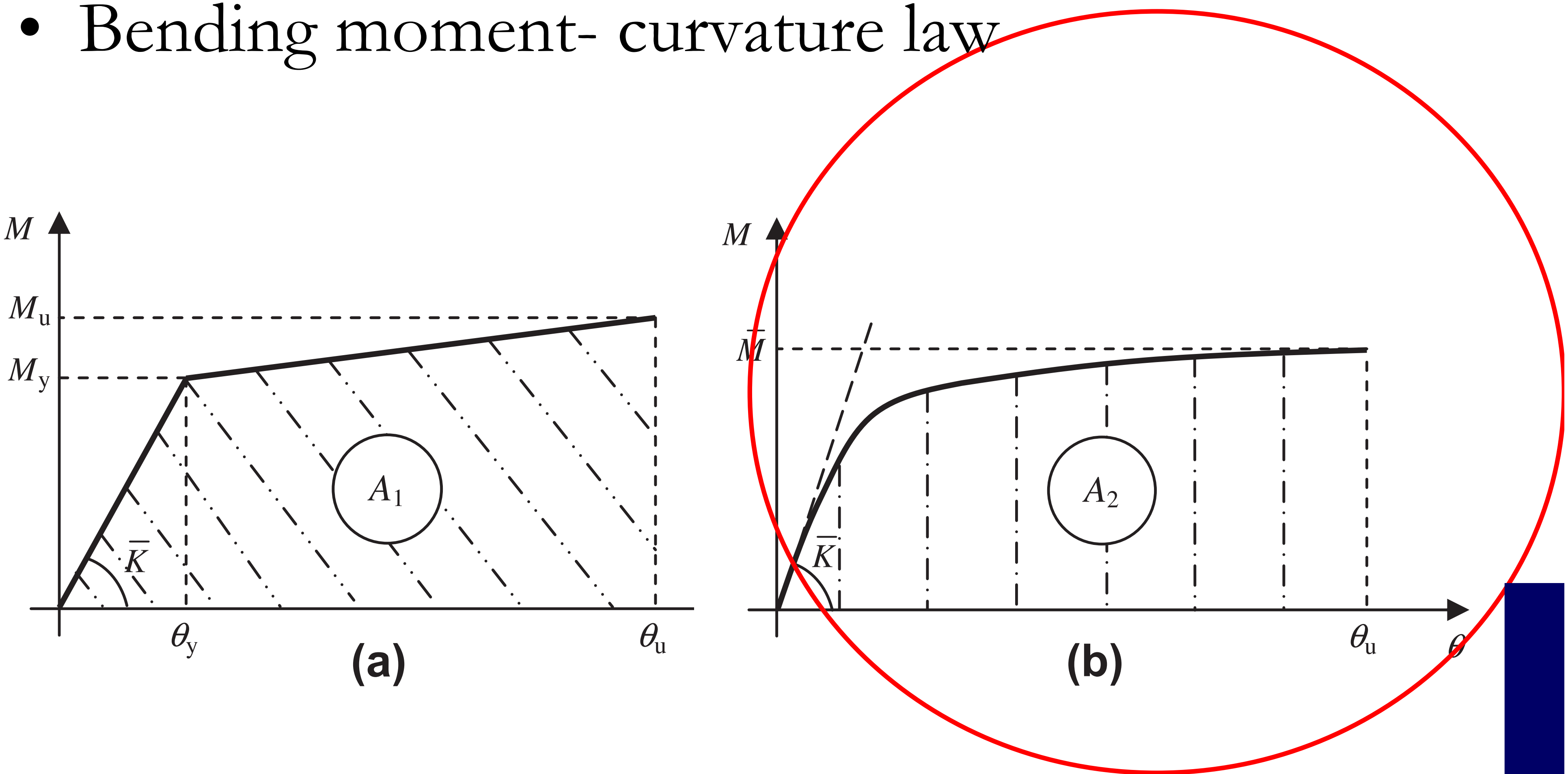


Bilinear

$$M = \bar{M}\tanh\left(\frac{\bar{K}}{\bar{M}}\theta\right) = -\bar{M}\tanh\left(\frac{\bar{K}}{\bar{M}}\frac{\partial^2 v}{\partial x^2}\right)$$

Dynamic model 1: continuous beam

- Bending moment- curvature law



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Dynamic model 1: continuous beam

- Equation of motion:

$$\frac{\partial^2 M}{\partial x^2} + q = \mu \frac{\partial^2 v}{\partial t^2}$$

$$M = \bar{M} \tanh\left(\frac{\bar{K}}{\bar{M}}\theta\right) = -\bar{M} \tanh\left(\frac{\bar{K}}{\bar{M}} \frac{\partial^2 v}{\partial x^2}\right)$$

$$\bar{K}(t)\,\mathrm{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)} \frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)} \tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)} \frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2 + \frac{\partial^4 v(x,t)}{\partial x^4}\right] + \mu \frac{\partial^2 v(x,t)}{\partial t^2} = q(x,t)$$

Dynamic model 1: continuous beam

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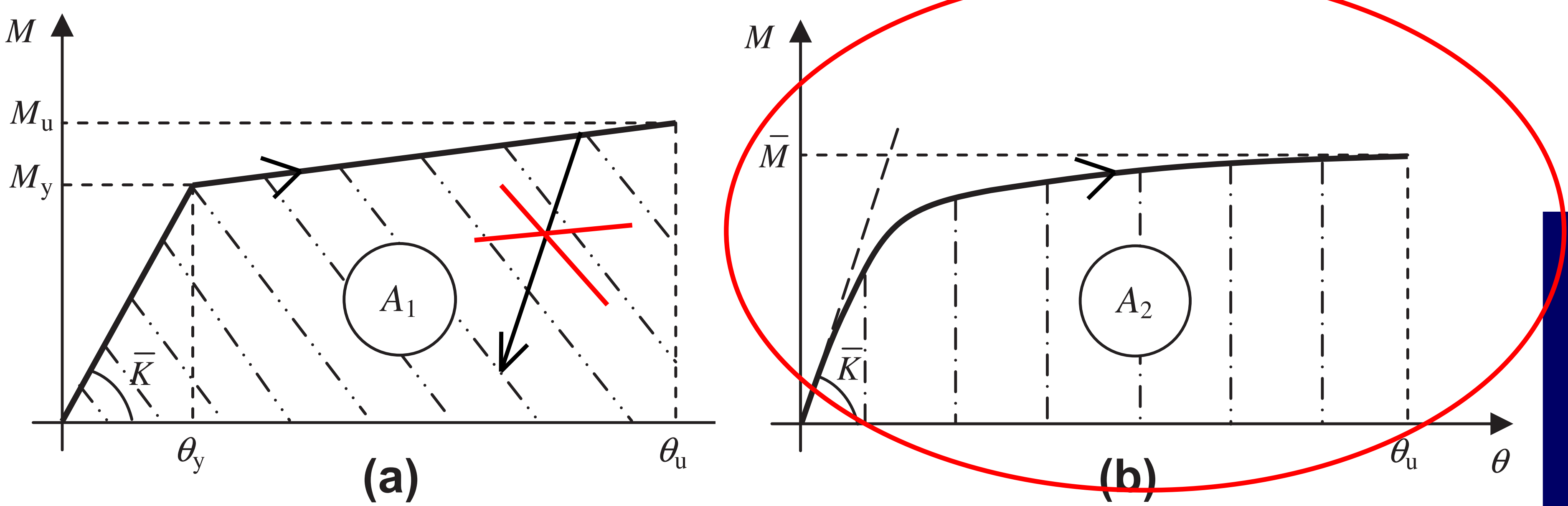
Dynamic model 1: continuous beam

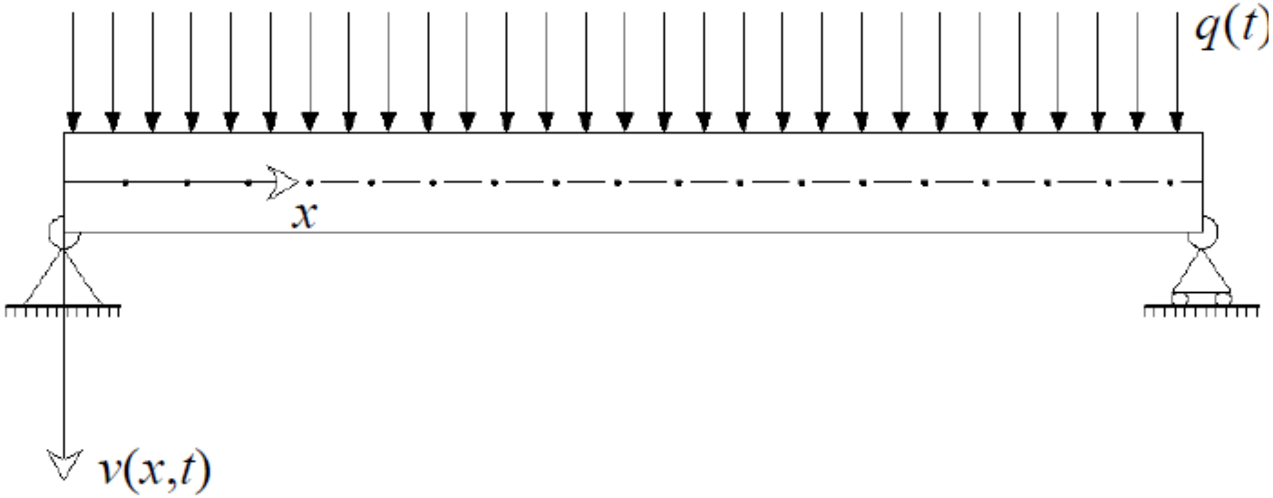
Simplified hypothesis: 1

Since the time when the impulsive load is applied is generally much smaller than the oscillation period of the beam, the failure of the beam usually occurs before the first peak of oscillation. We'll assume monotone constitutive laws.



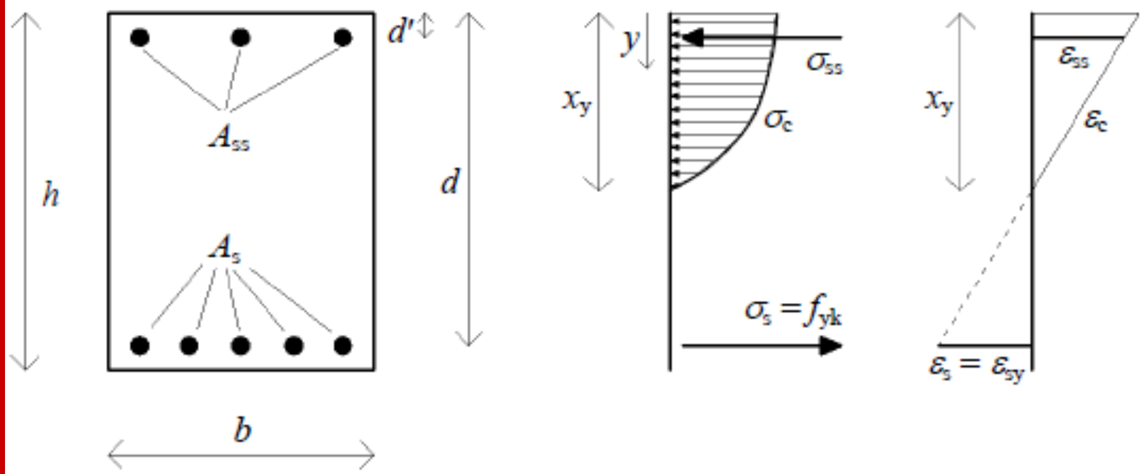
Dynamic model 1: continuous beam

Simplified hypothesis: 2



Flexural Failure

Dynamic model 1: continuous beam



Yielding limit characteristics M,x,θ

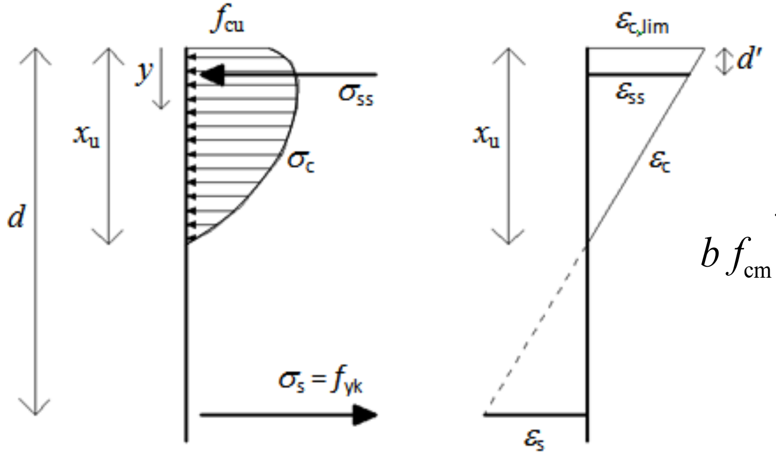
Translational Equilibrium:

$$b\int_0^{x_y} \sigma_c \, dy + \sigma_{ss} A_{ss} = f_{yk} A_s \qquad b\, f_{cm} \int_0^{x_y} \left[\frac{k \dfrac{\varepsilon_{sy}}{\varepsilon_{c1}} \dfrac{x_y - y}{d - x_y} - \left(\dfrac{\varepsilon_{sy}}{\varepsilon_{c1}} \dfrac{x_y - y}{d - x_y} \right)^2}{1 + (k-2) \dfrac{\varepsilon_{sy}}{\varepsilon_{c1}} \dfrac{x_y - y}{d - x_y}} \right] dy + E_s \, \varepsilon_{sy} \frac{x_y - d'}{d - x_y} A_{ss} = f_{yk} A_s$$

Rotational equilibrium:

$$M_y = b\int_0^{x_y} \sigma_c (d - y) \, dy + \sigma_{ss} A_{ss} (d - d') = b\, f_{cm} \int_0^{x_y} \left[\frac{k \dfrac{\varepsilon_{sy}}{\varepsilon_{c1}} \dfrac{x_y - y}{d - x_y} - \left(\dfrac{\varepsilon_{sy}}{\varepsilon_{c1}} \dfrac{x_y - y}{d - x_y} \right)^2}{1 + (k-2) \dfrac{\varepsilon_{sy}}{\varepsilon_{c1}} \dfrac{x_y - y}{d - x_y}} \right] (d - y) \, dy + E_s \, \varepsilon_{sy} \frac{x_y - d'}{d - x_y} A_{ss} (d - d')$$

Dynamic model 1: continuous beam



Ultimate limit characteristics M,x,θ

Translational Equilibrium

$$b\,f_{cm}\int_0^{x_u}\left[\frac{k\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}-\left(\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}\right)^2}{1+(k-2)\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}}\right]dy+E_s\,\varepsilon_{c,lim}\frac{x_u-d'}{x_u}A_{ss}=f_{yk}A_s \quad \text{se } \sigma_{ss}<f_{yk}$$

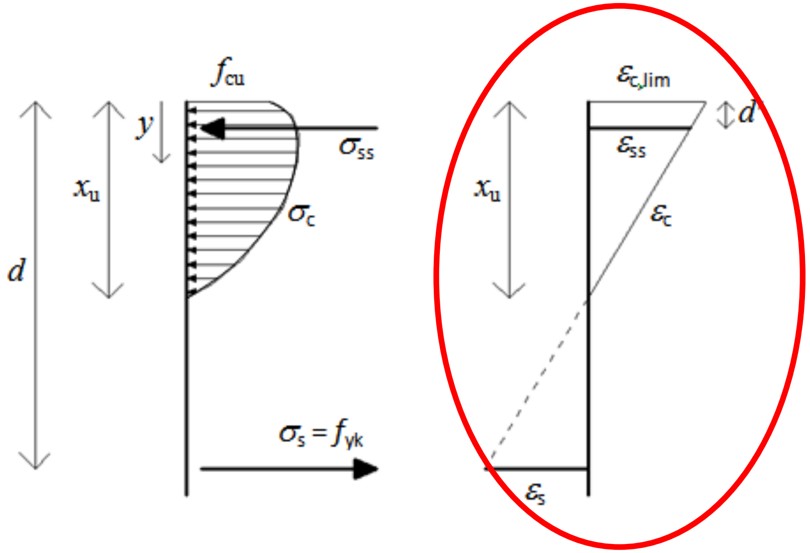
$$b\,f_{cm}\int_0^{x_u}\left[\frac{k\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}-\left(\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}\right)^2}{1+(k-2)\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}}\right]dy+f_{yk}A_{ss}=f_{yk}A_s \quad \text{se } \sigma_{ss}\geq f_{yk}$$

Rotational equilibrium

$$M_u=b\,f_{cm}\int_0^{x_u}\left[\frac{k\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}-\left(\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}\right)^2}{1+(k-2)\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}}\right](d-y)\,dy+E_s\,\varepsilon_{c,lim}\frac{x_u-d'}{x_u}A_{ss}(d-d') \quad \text{se } \sigma_{ss}<f_{yk}$$

$$M_u=b\,f_{cm}\int_0^{x_u}\left[\frac{k\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}-\left(\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}\right)^2}{1+(k-2)\dfrac{\varepsilon_{c,lim}}{\varepsilon_{c1}}\dfrac{x_u-y}{x_u}}\right](d-y)\,dy+f_{yk}\,A_{ss}(d-d') \quad \text{se } \sigma_{ss}\geq f_{yk}$$

Dynamic model 1: continuous beam



Collapse Criterium

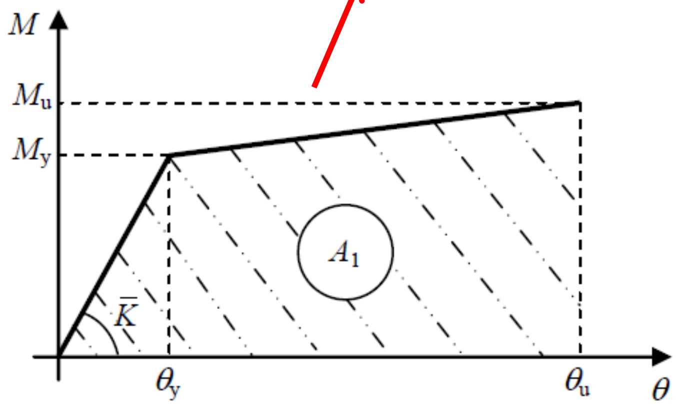
Dynamic model 1: continuous beam

Given M_u, M_y, x_u, x_y it is easy to calculate $\bar{K}$ and $\bar{M}$.

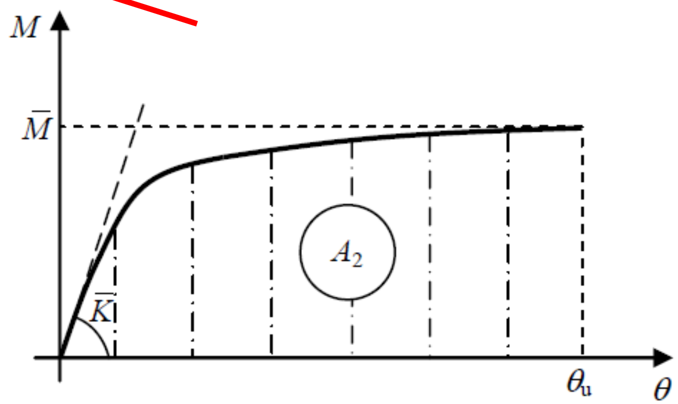
$$\theta_y = \frac{\varepsilon_{sy}}{d - x_y} \qquad \theta_u = \frac{\varepsilon_{c,lim}}{x_u}$$

$$\frac{\bar{M}^2}{\bar{K}} \ln\left[\cosh\left(\frac{\bar{K}}{\bar{M}}\theta_u\right)\right] = \frac{M_u(\theta_u - \theta_y) + M_y\theta_u}{2}$$

$$\bar{K} = \frac{M_y}{\theta_y}$$



(a)



(b)

Dynamic model 1: continuous beam

- Strain Rate Effect CEB Bulletin 187 §3.4.2 - §3.31

$$f_{\mathrm{cm,dyn}} = f_{\mathrm{cm}} \cdot \left(\frac{\dot{\varepsilon}_{\mathrm{c}}}{30 \cdot 10^{-6}} \right)^{1.026 \cdot \alpha} \quad \text{if } \dot{\varepsilon}_{\mathrm{c}} \leq 30\,\mathrm{s}^{-1}$$

$$f_{\mathrm{cm,dyn}} = f_{\mathrm{cm}} \cdot \gamma \cdot (\dot{\varepsilon}_{\mathrm{c}})^{1/3} \quad \text{if } \dot{\varepsilon}_{\mathrm{c}} > 30\,\mathrm{s}^{-1}$$

$$\varepsilon_{\mathrm{c1,dyn}} = \varepsilon_{\mathrm{c1}} \cdot \left(\frac{\dot{\varepsilon}_{\mathrm{c}}}{30 \cdot 10^{-6}} \right)^{0.02}$$

$$\varepsilon_{\mathrm{c,lim,dyn}} = \varepsilon_{\mathrm{c,lim}} \cdot \left(\frac{\dot{\varepsilon}_{\mathrm{c}}}{30 \cdot 10^{-6}} \right)^{0.02}$$

$$f_{\mathrm{yk,dyn}} = f_{\mathrm{yk}} \cdot \left[1 + \frac{6}{f_{\mathrm{yk}}} \ln \left(\frac{\dot{\varepsilon}^{\mathrm{s}}}{5 \cdot 10^{-5}} \right) \right] \quad \text{if } \dot{\varepsilon}^{\mathrm{s}} \leq 10\,\mathrm{s}^{-1}$$

Dynamic model 1: continuous beam

The equation of motion is a nonlinear PDE with variable coefficients. Its solution can be obtained by means of a numerical approach. An iterative procedure is performed, which consists in evaluating at each time step the following quantities:

$$\bar{K}(t)\,\mathrm{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

1. the vertical displacement v, which is obtained by solving the equation of motion where $\bar{K}$ and $\bar{M}$ are varied at each time step due to strain rate effects;

Dynamic model 1: continuous beam

Numerical discretization:

$$\bar{K}(t)\operatorname{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

$$\frac{\partial^2 v(x,t)}{\partial x^2}=\frac{v_{(i-1,j)}-2v_{(i\ \ ,j)}+v_{(i+1,j)}}{h^2}$$

$$\frac{\partial^3 v(x,t)}{\partial x^3}=\frac{-v_{(i-2,j)}+2v_{(i-1,j)}-2v_{(i+1,j)}+v_{(i+2,j)}}{2h^3}$$

$$\frac{\partial^4 v(x,t)}{\partial x^4}=\frac{-v_{(i-2,j)}-4v_{(i-1,j)}+6v_{(i,j)}-4v_{(i+1,j)}+v_{(i+2,j)}}{h^4}$$

$$\frac{\partial^2 v(x,t)}{\partial t^2}=\frac{v_{(i,j-1)}-2v_{(i,j)}+v_{(i,j+1)}}{k^2}$$

2nd order Finite Difference

Dynamic model 1: continuous beam

$$\bar{K}(t)\operatorname{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

Numerical Integration Scheme:

$$K_{(j)}\operatorname{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{v_{(i-1,j)}-2v_{(i,j)}+v_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l}-2\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{v_{(i-1,j)}-2v_{(i,j)}+v_{(i+1,j)}}{h^2}\right)\times\\ \times\left(\frac{-v_{(i-2,j)}+2v_{(i-1,j)}-2v_{(i+1,j)}+v_{(i+2,j)}}{h^3}\right)^2+\\ +\frac{v_{(i-2,j)}-4v_{(i-1,j)}+6v_{(i,j)}-4v_{(i+1,j)}+v_{(i+2,j)}}{h^4}\end{array}\right]+$$

$$+\mu\frac{v_{(i,j-1)}-2v_{(i,j)}+v_{(i,j+1)}}{k^2}=q(i,j)$$

Dynamic model 1: continuous beam

$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{v_{(i-1,j)}-2v_{(i,j)}+v_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l} -2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{v_{(i-1,j)}-2v_{(i,j)}+v_{(i+1,j)}}{h^2}\right)\times \\ \times\left(\dfrac{-v_{(i-2,j)}+2v_{(i-1,j)}-2v_{(i+1,j)}+v_{(i+2,j)}}{h^3}\right)^2+ \\ +\dfrac{v_{(i-2,j)}-4v_{(i-1,j)}+6v_{(i,j)}-4v_{(i+1,j)}+v_{(i+2,j)}}{h^4} \end{array}\right]+$$

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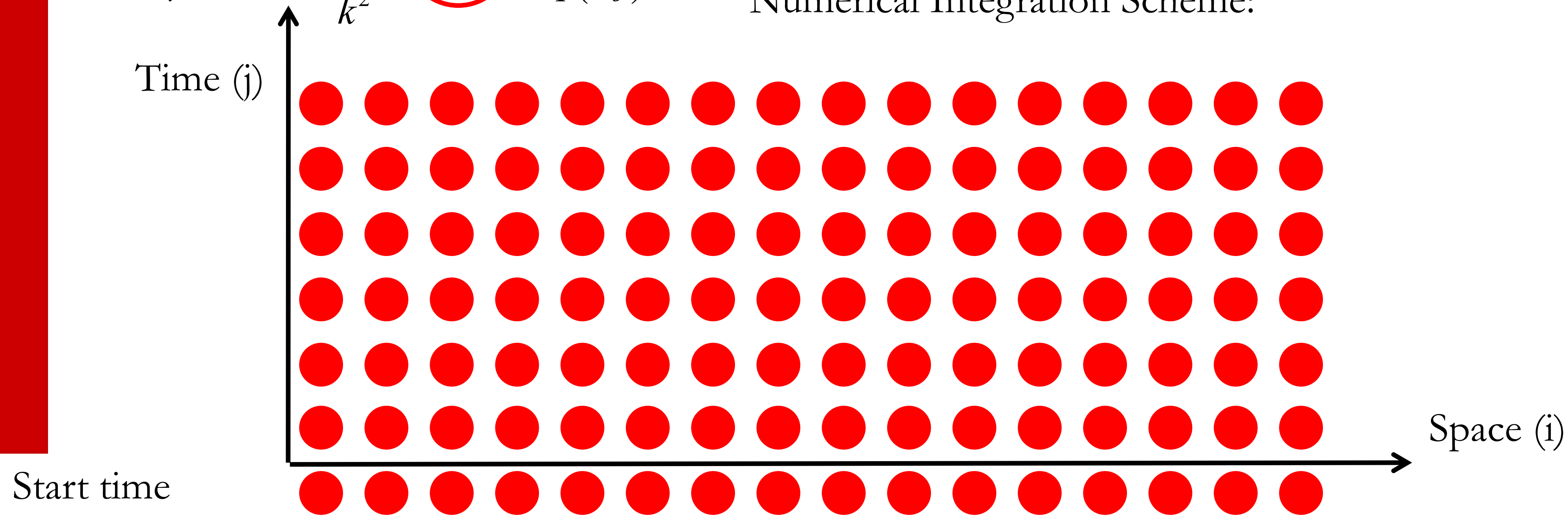

Dynamic model 1: continuous beam

$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l}-2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times\\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+\\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}\end{array}\right]+$$

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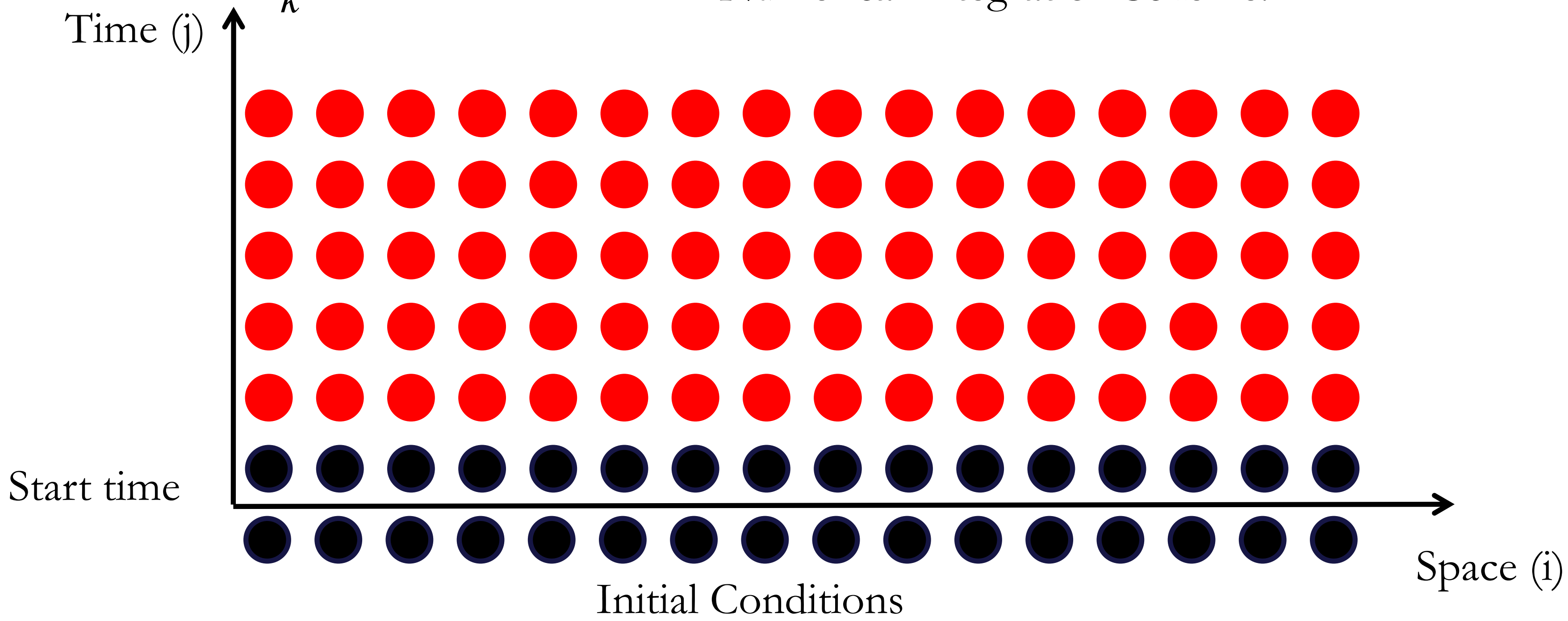
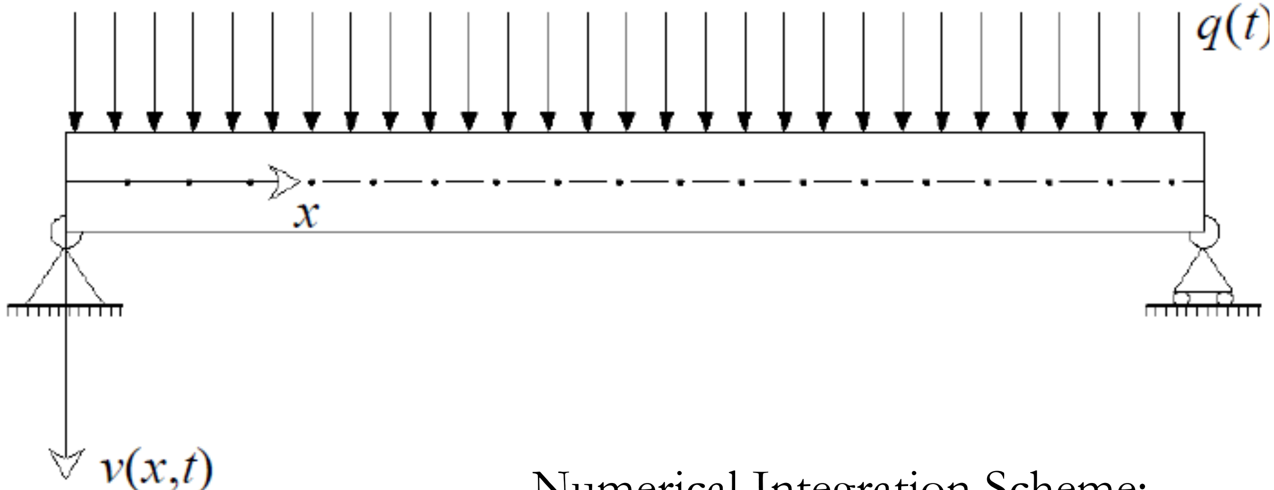
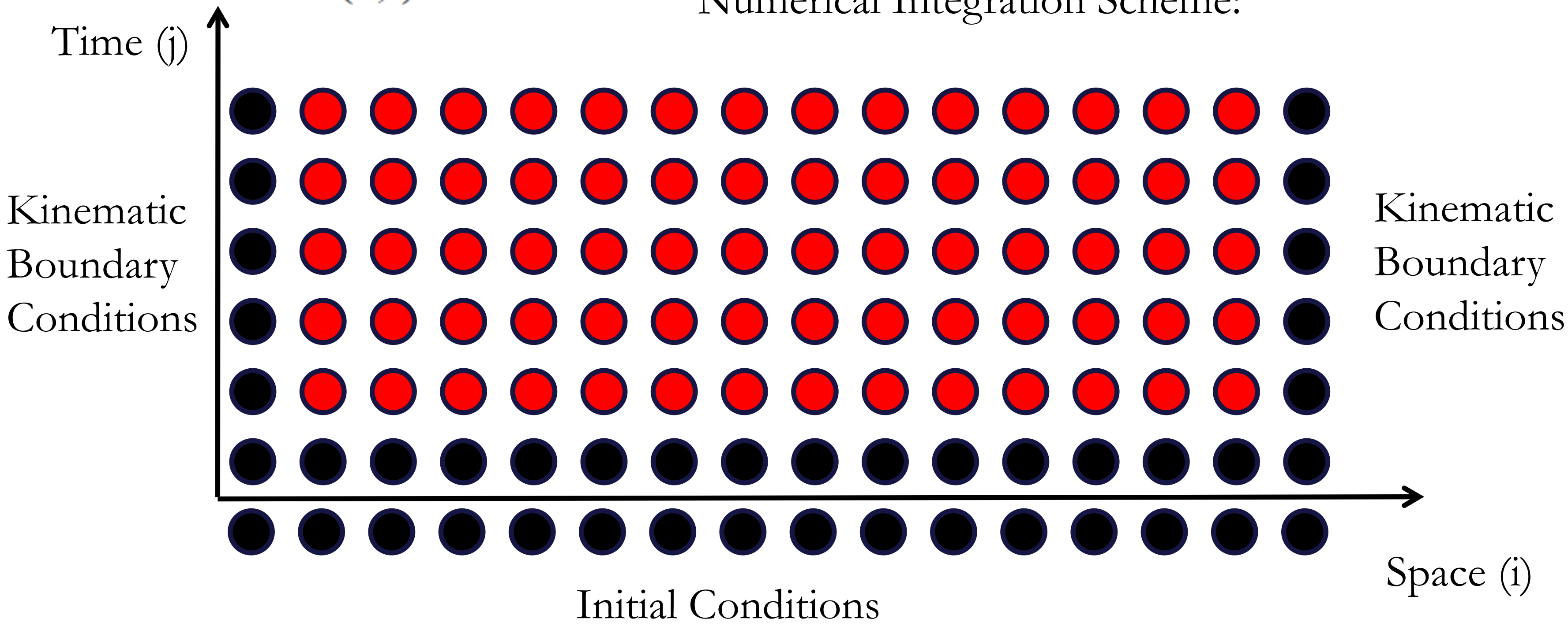


Dynamic model 1: continuous beam



Numerical Integration Scheme:



Dynamic model 1: continuous beam

$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\,\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l} -2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\,\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times \\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+ \\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4} \end{array}\right]+$$

$$+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q(i,j)$$

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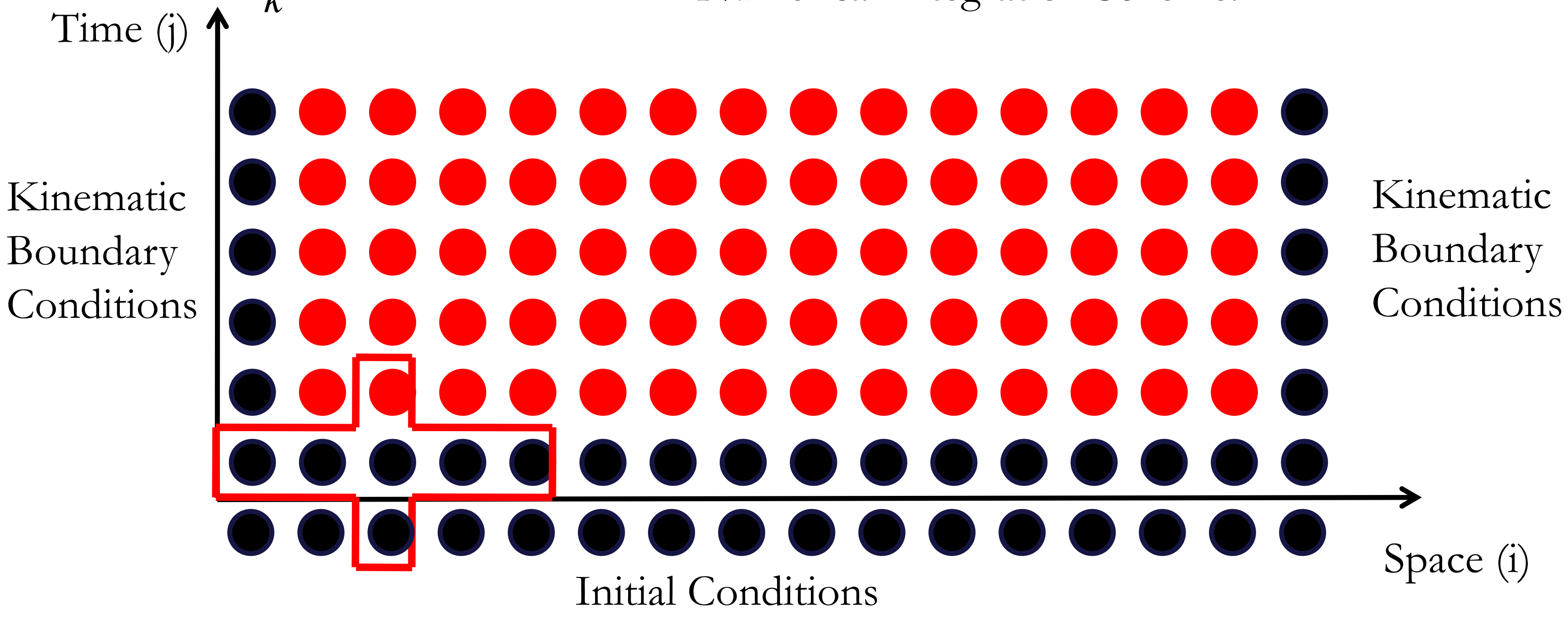

Dynamic model 1: continuous beam

$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l}-2\,\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times\\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+\\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}\end{array}\right]+$$

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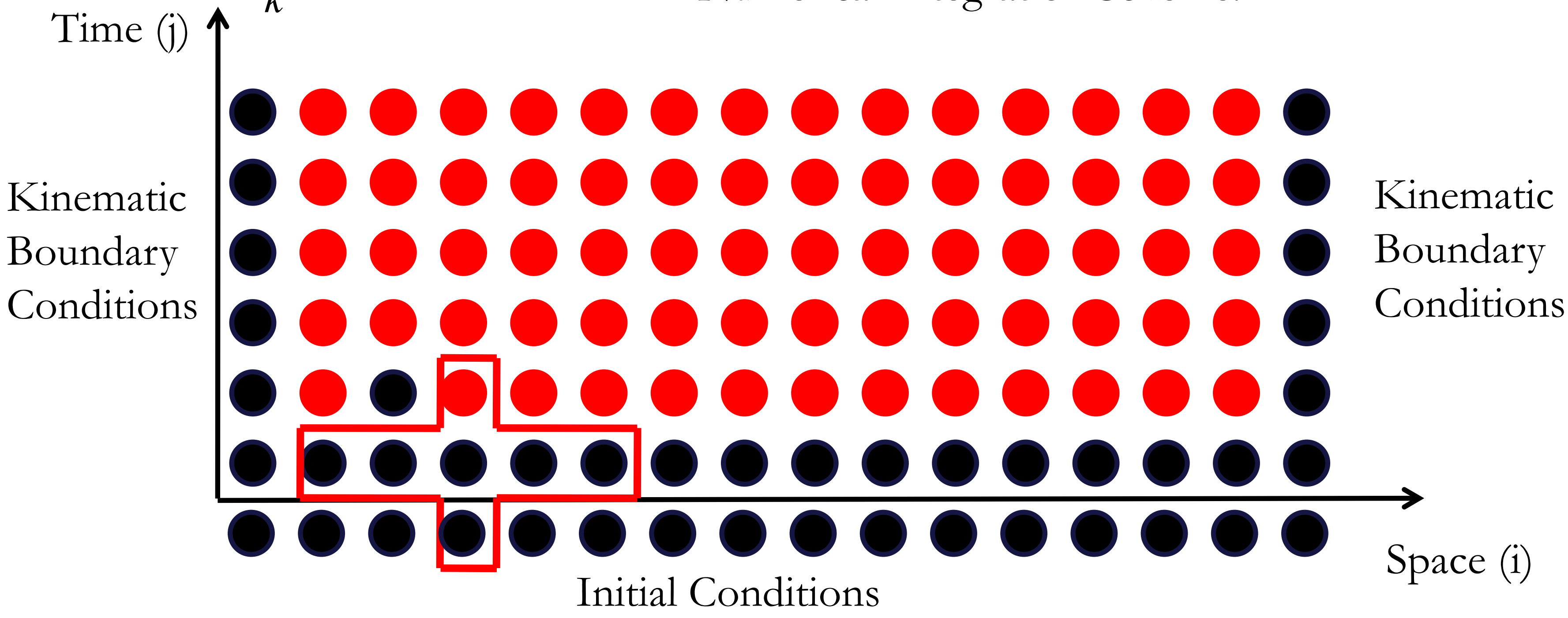
Numerical Integration Scheme:

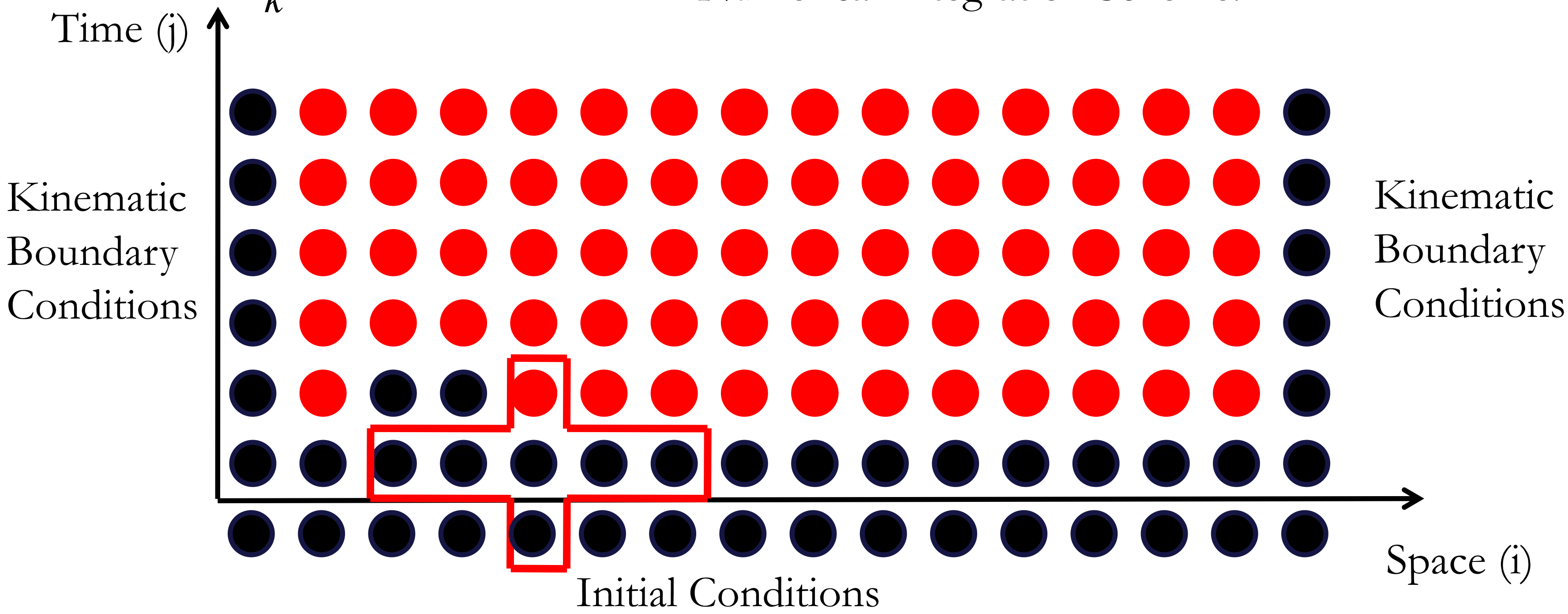


Dynamic model 1: continuous beam

$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l} -2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times \\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+ \\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4} \end{array}\right]+$$

$$+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q(i,j)$$

Numerical Integration Scheme:

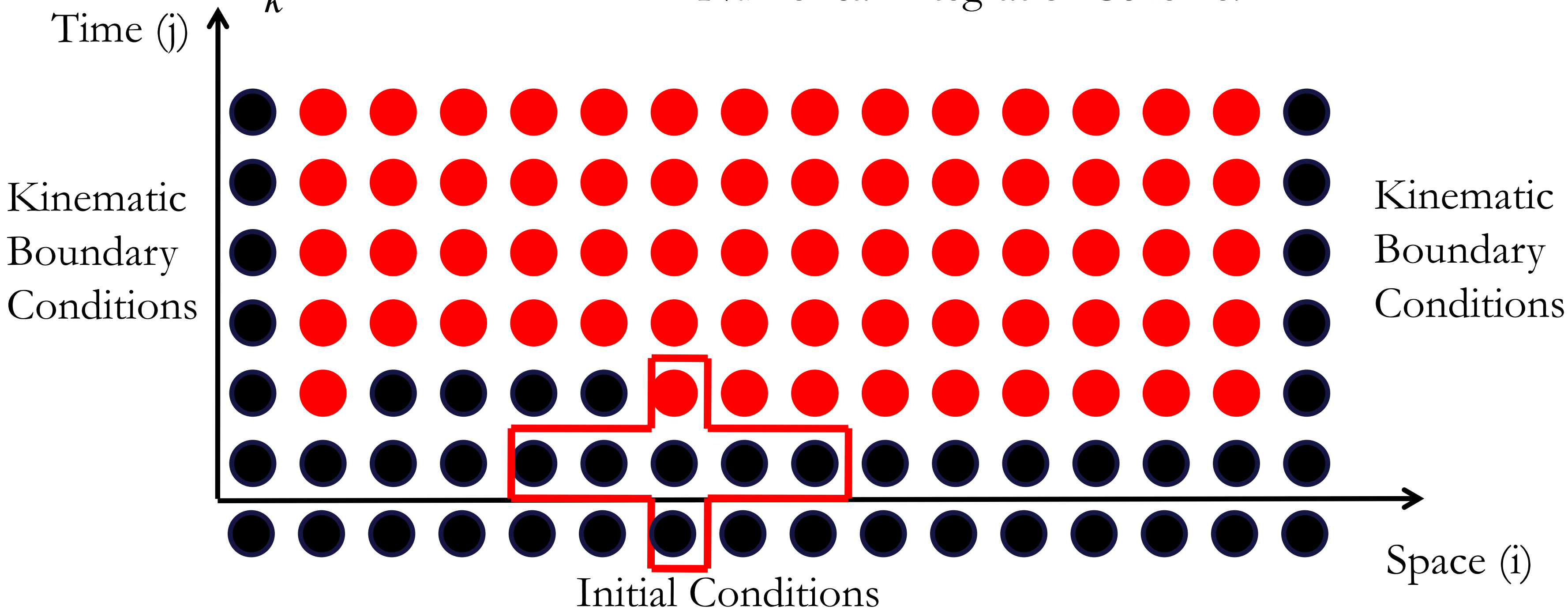


Dynamic model 1: continuous beam

$$
K_{(j)}\,\mathrm{sech}^2\!\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)
\left[\begin{aligned}
&-2\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times\\
&\times\left(\frac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+\\
&+\frac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}
\end{aligned}\right]+
$$

$$
+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q(i,j)
$$

Numerical Integration Scheme:

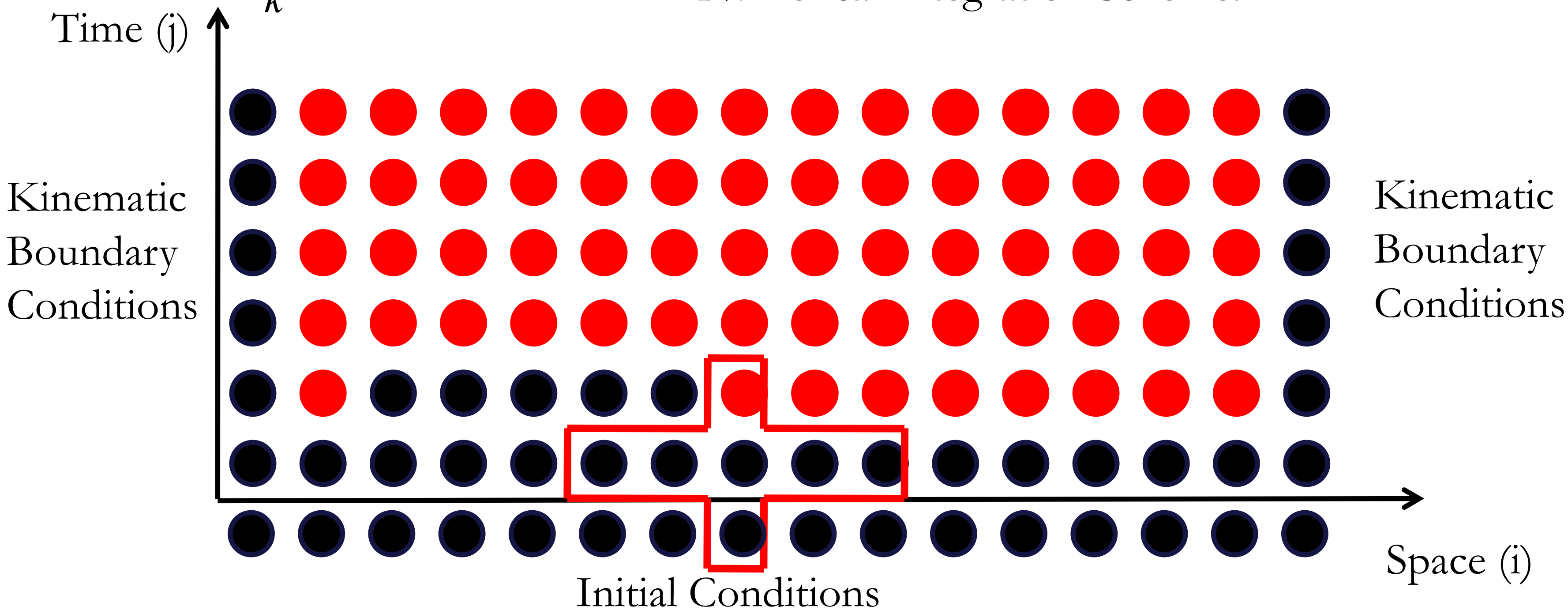


Dynamic model 1: continuous beam

$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l}-2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times\\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+\\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}\end{array}\right]+$$

$$+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q(i,j)$$

Numerical Integration Scheme:

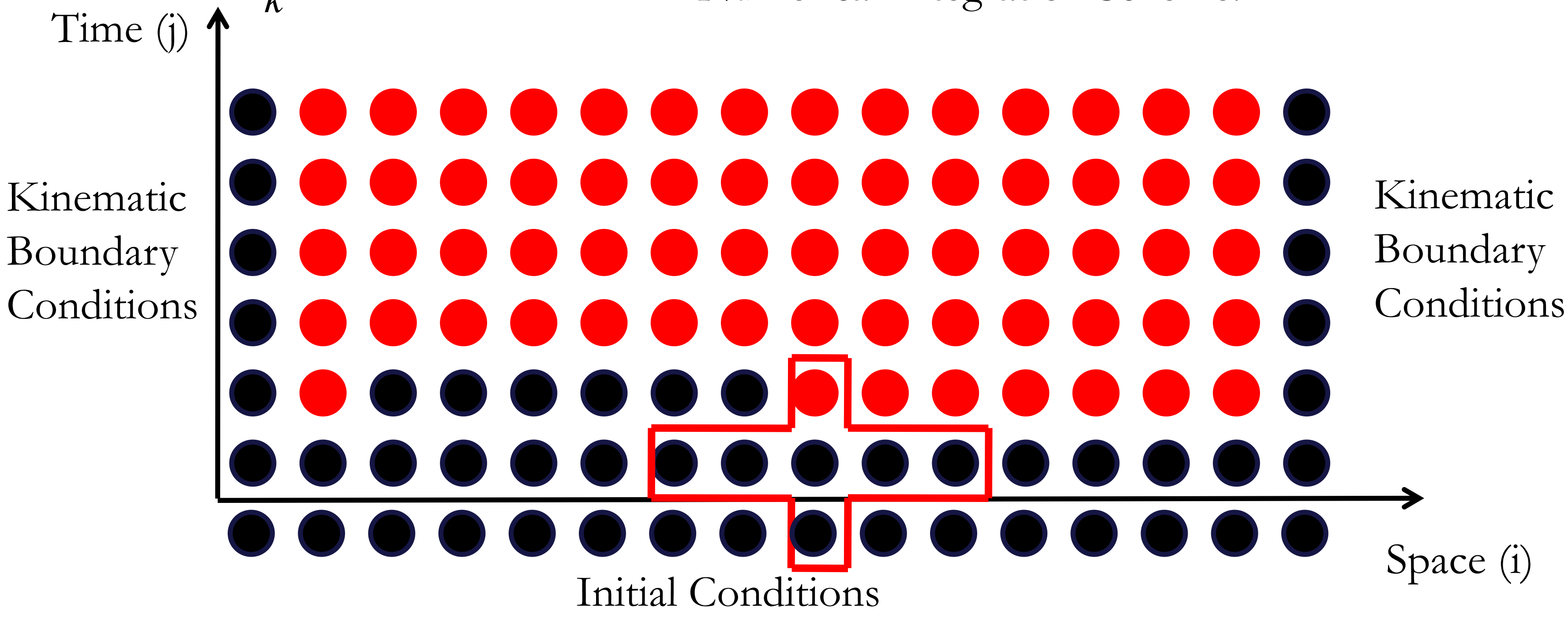


Dynamic model 1: continuous beam

$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l} -2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times \\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+ \\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4} \end{array}\right]+$$

$$+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q(i,j)$$

Numerical Integration Scheme:



Dynamic model 1: continuous beam

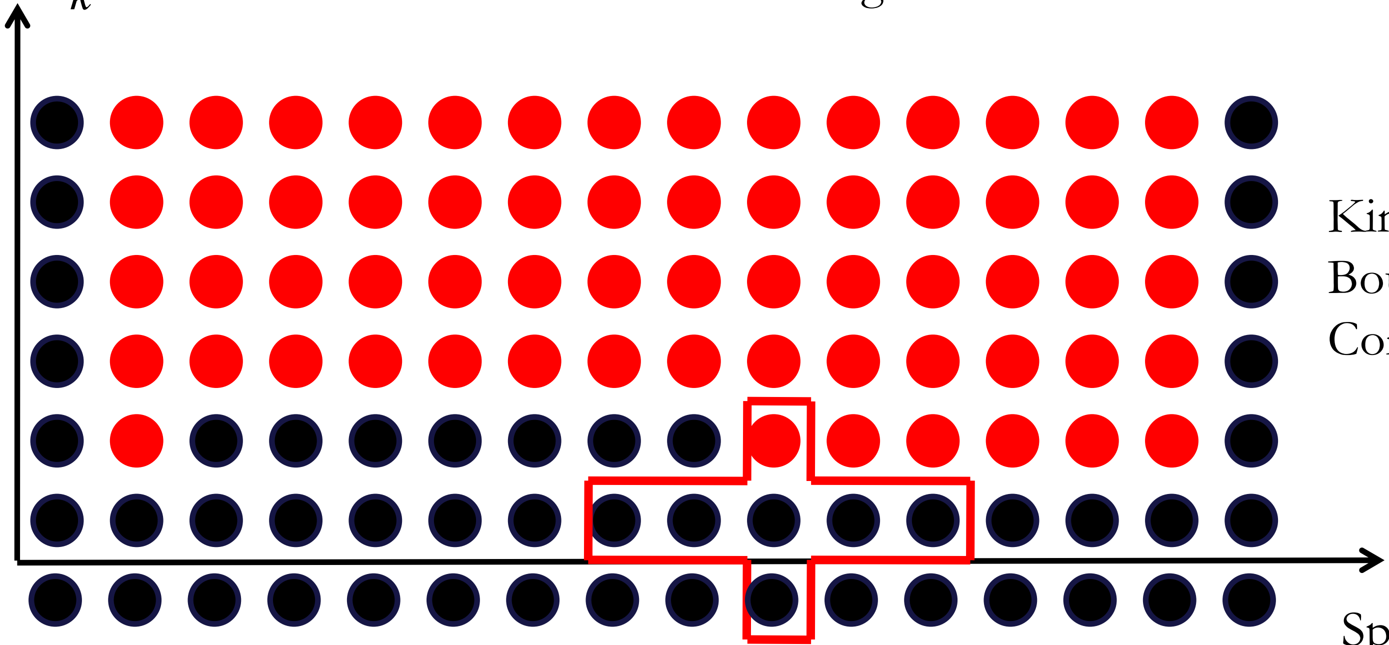
$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l}-2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times\\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+\\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}\end{array}\right]+$$

$$+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q(i,j)$$

Numerical Integration Scheme:

Time (j)

Kinematic Boundary Conditions



Kinematic Boundary Conditions

Space (i)

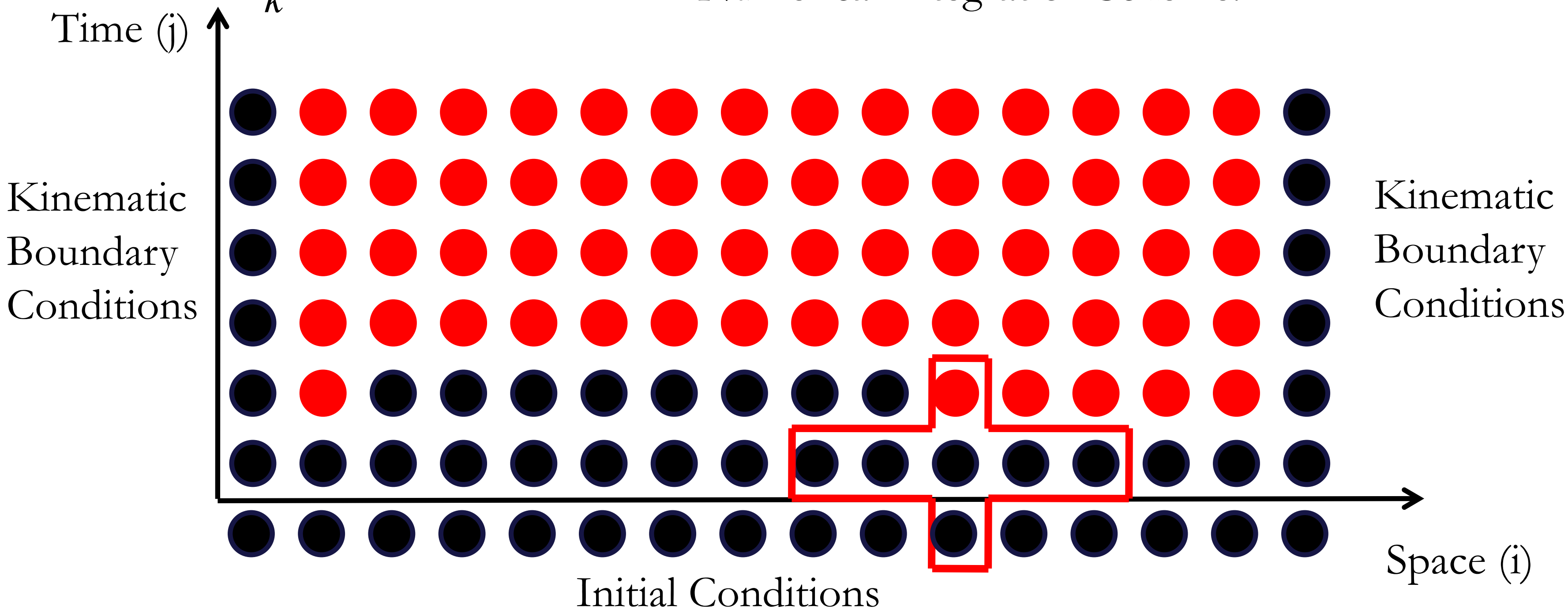
Initial Conditions

Dynamic model 1: continuous beam

$$
K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)
\left[\begin{array}{l}
-2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times \\
\times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2 + \\
+\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}
\end{array}\right]+
$$

$$
+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q(i,j)
$$

Numerical Integration Scheme:

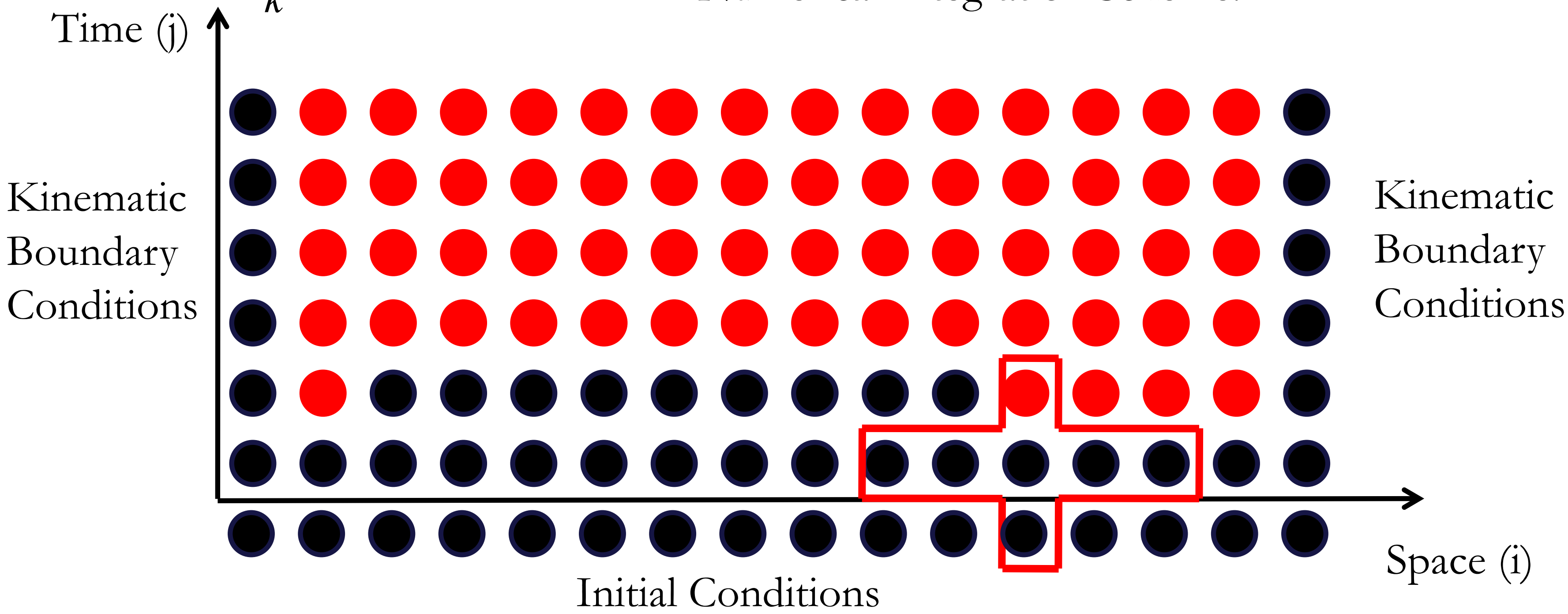


Dynamic model 1: continuous beam

$$
K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l}-2\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times\\ \times\left(\frac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+\\ +\frac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}\end{array}\right]+
$$

$$
+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q\,(i,j)
$$

Numerical Integration Scheme:

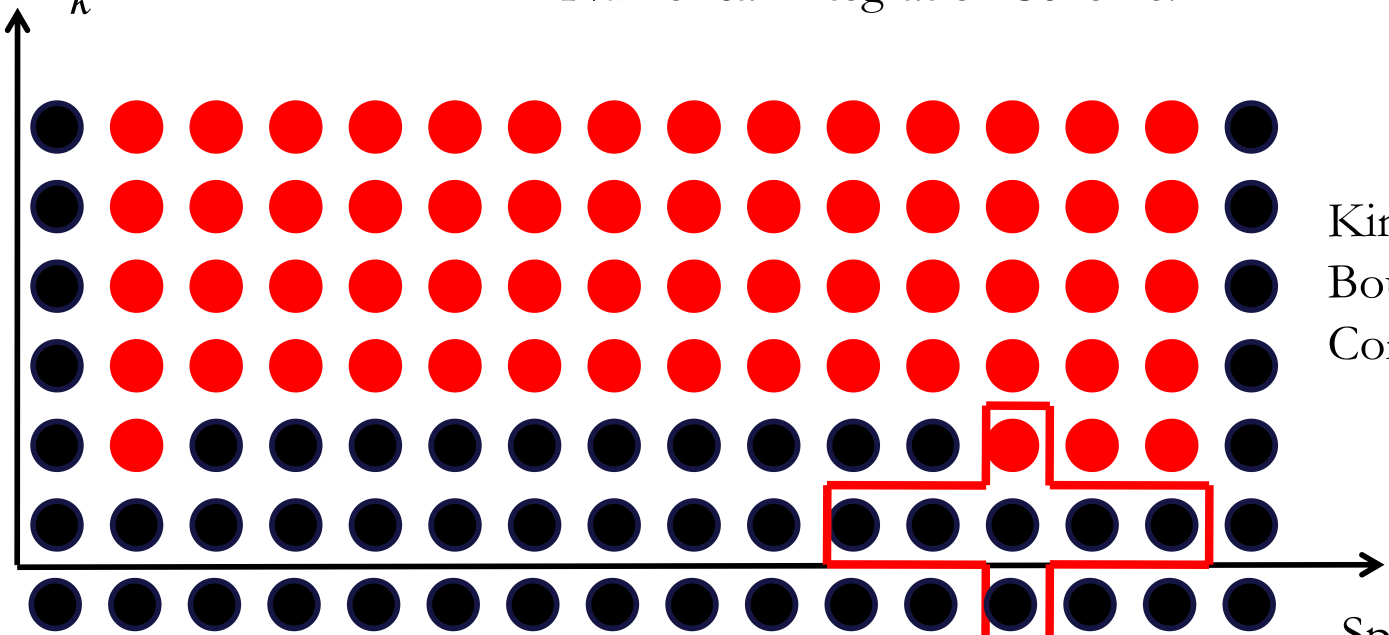


Dynamic model 1: continuous beam

$$
K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)
\left[\begin{array}{l}
-2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times \\
\times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+ \\
+\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}
\end{array}\right]+
$$

$$
+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q(i,j)
$$

Numerical Integration Scheme:

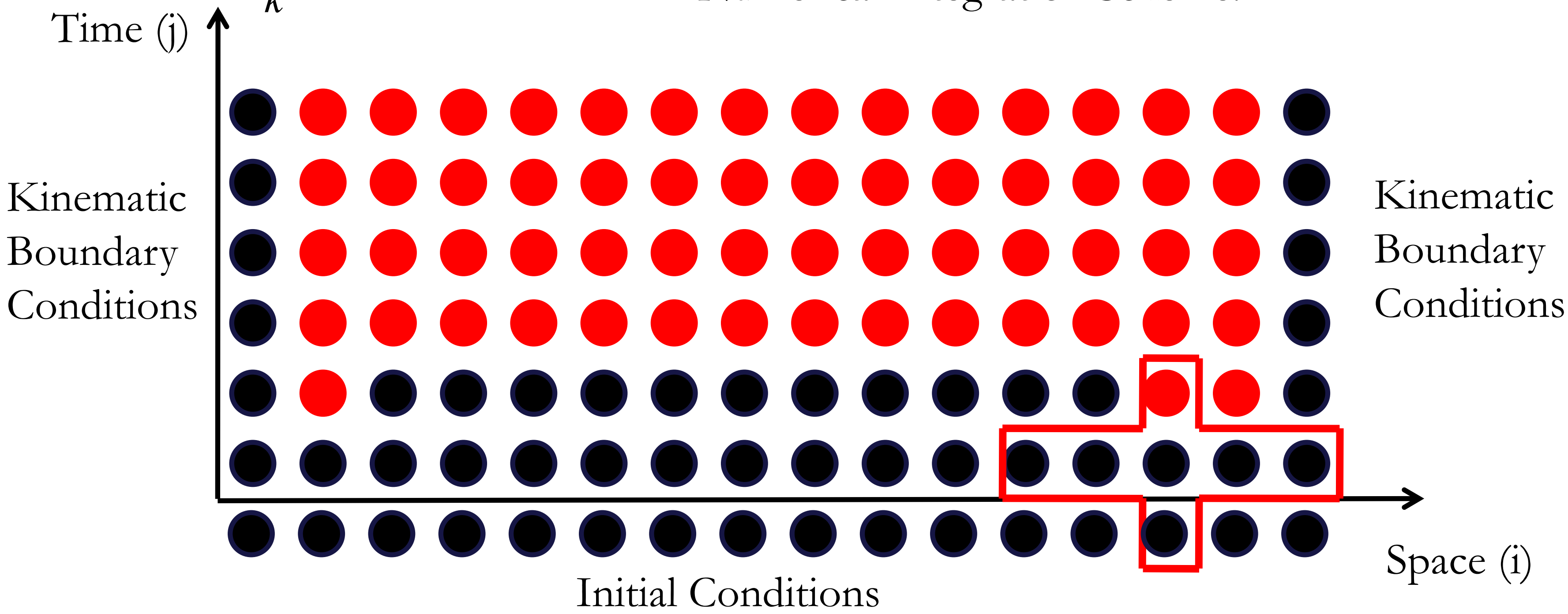


Dynamic model 1: continuous beam

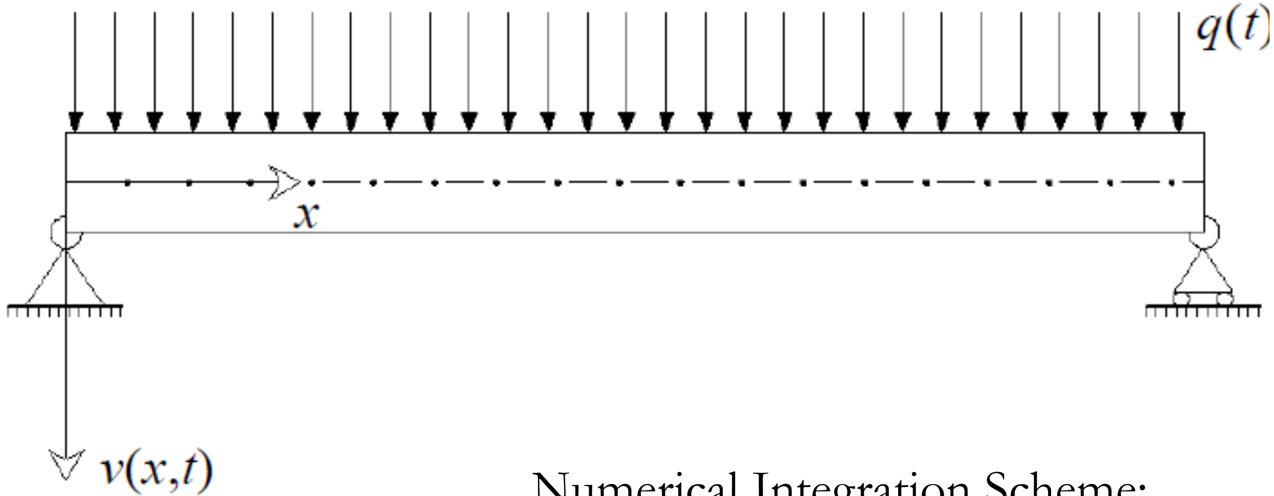
$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l}-2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times\\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+\\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}\end{array}\right]+$$

$$+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q\,(i,j)$$

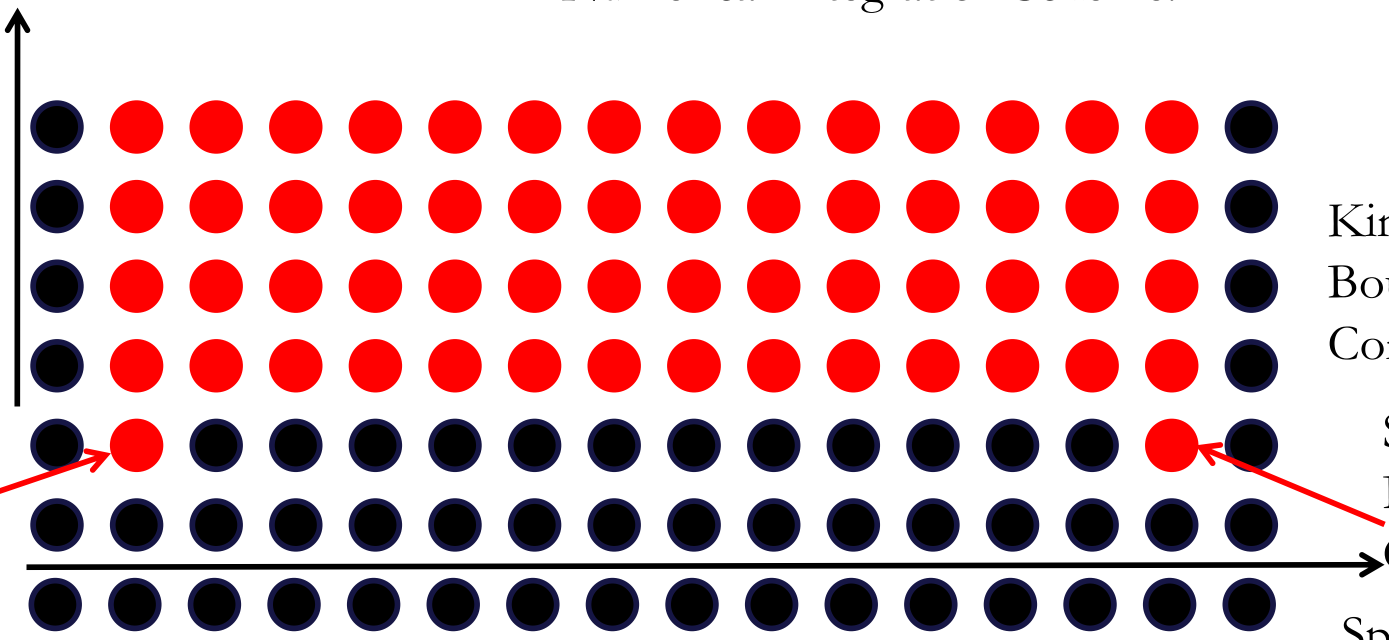
Numerical Integration Scheme:



Dynamic model 1: continuous beam



Numerical Integration Scheme:

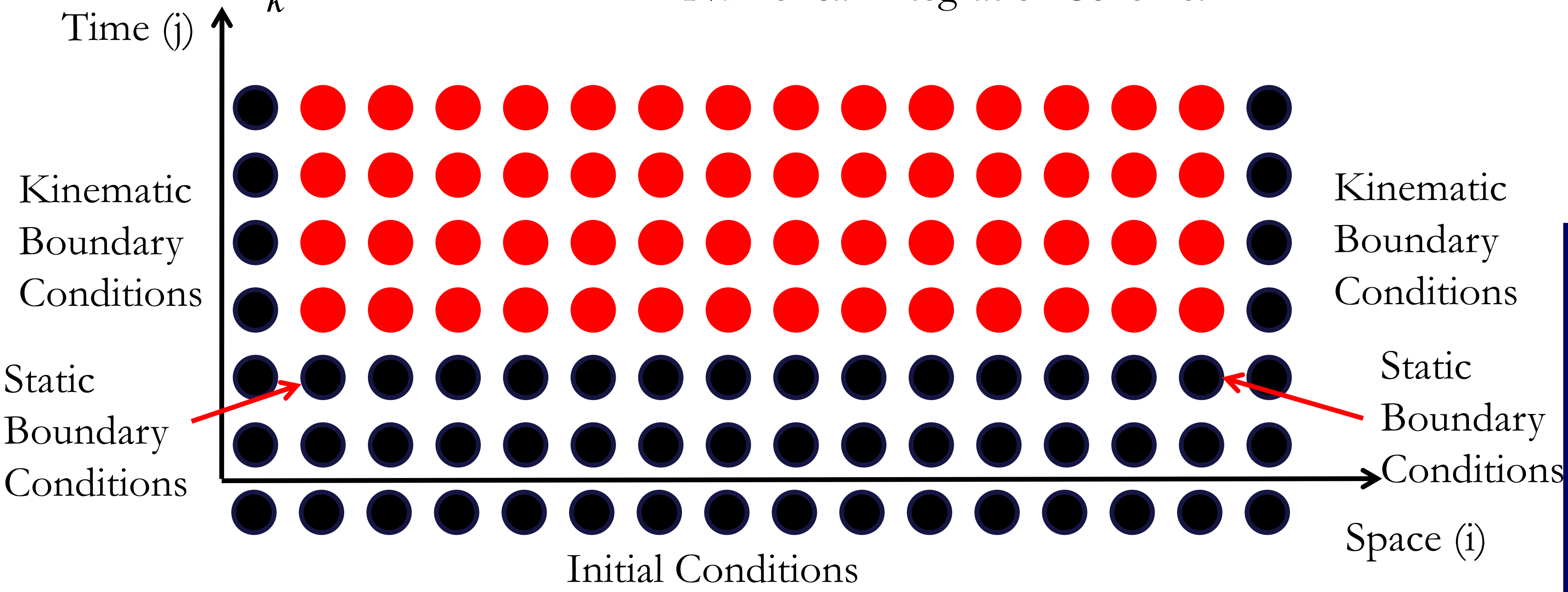


Dynamic model 1: continuous beam

$$K_{(j)}\,\mathrm{sech}^2\left(\frac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\frac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\left[\begin{array}{l}-2\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\tanh\left(\dfrac{\bar{K}_{(j)}}{\bar{M}_{(j)}}\dfrac{u_{(i-1,j)}-2u_{(i,j)}+u_{(i+1,j)}}{h^2}\right)\times\\ \times\left(\dfrac{-u_{(i-2,j)}+2u_{(i-1,j)}-2u_{(i+1,j)}+u_{(i+2,j)}}{h^3}\right)^2+\\ +\dfrac{u_{(i-2,j)}-4u_{(i-1,j)}+6u_{(i,j)}-4u_{(i+1,j)}+u_{(i+2,j)}}{h^4}\end{array}\right]+$$

$$+\mu\frac{u_{(i,j-1)}-2u_{(i,j)}+u_{(i,j+1)}}{k^2}=q\,(i,j)$$

Numerical Integration Scheme:



Dynamic model 1: continuous beam

The equation of motion is a nonlinear PDE with variable coefficients. Its solution can be obtained by means of a numerical approach. An iterative procedure is performed, which consists in evaluating at each time step the following quantities:

$$\bar{K}(t)\,\mathrm{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

1. the vertical displacement v, which is obtained by solving the equation of motion where $\bar{K}$ and $\bar{M}$ are varied at each time step due to strain rate effects;
2. the curvature $\theta = -\partial^2 v/\partial x^2$ and the rate of curvature $\partial\theta/\partial t$;

Dynamic model 1: continuous beam

The equation of motion is a nonlinear PDE with variable coefficients. Its solution can be obtained by means of a numerical approach. An iterative procedure is performed, which consists in evaluating at each time step the following quantities:

$$\bar{K}(t)\operatorname{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

1. the vertical displacement v, which is obtained by solving the equation of motion where $\bar{K}$ and $\bar{M}$ are varied at each time step due to strain rate effects;
2. the curvature $\theta = -\partial^2 v/\partial x^2$ and the rate of curvature $\partial\theta/\partial t$;
3. the bending moment M corresponding to the curvature θ;

Dynamic model 1: continuous beam

The equation of motion is a nonlinear PDE with variable coefficients. Its solution can be obtained by means of a numerical approach. An iterative procedure is performed, which consists in evaluating at each time step the following quantities:

$$\bar{K}(t)\,\mathrm{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

1. the vertical displacement v, which is obtained by solving the equation of motion where $\bar{K}$ and $\bar{M}$ are varied at each time step due to strain rate effects;
2. the curvature $\theta = -\partial^2 v/\partial x^2$ and the rate of curvature $\partial\theta/\partial t$;
3. the bending moment M corresponding to the curvature θ;
4. the neutral axis depth from rotational equilibrium around the tensile reinforcement under the applied bending moment M;

Dynamic model 1: continuous beam

The equation of motion is a nonlinear PDE with variable coefficients. Its solution can be obtained by means of a numerical approach. An iterative procedure is performed, which consists in evaluating at each time step the following quantities:

$$\bar{K}(t)\,\mathrm{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

1. the vertical displacement v, which is obtained by solving the equation of motion where $\bar{K}$ and $\bar{M}$ are varied at each time step due to strain rate effects;
2. the curvature $\theta = -\partial^2 v/\partial x^2$ and the rate of curvature $\partial\theta/\partial t$;
3. the bending moment M corresponding to the curvature θ;
4. the neutral axis depth from rotational equilibrium around the tensile reinforcement under the applied bending moment M;
5. the strains of concrete and steel reinforcements by using the linear deformation diagram and the value of curvature;

Dynamic model 1: continuous beam

The equation of motion is a nonlinear PDE with variable coefficients. Its solution can be obtained by means of a numerical approach. An iterative procedure is performed, which consists in evaluating at each time step the following quantities:

$$\bar{K}(t)\,\mathrm{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

1. the vertical displacement v, which is obtained by solving the equation of motion where $\bar{K}$ and $\bar{M}$ are varied at each time step due to strain rate effects;
2. the curvature $\theta = -\partial^2 v/\partial x^2$ and the rate of curvature $\partial\theta/\partial t$;
3. the bending moment M corresponding to the curvature θ;
4. the neutral axis depth from rotational equilibrium around the tensile reinforcement under the applied bending moment M;
5. the strains of concrete and steel reinforcements by using the linear deformation diagram and the value of curvature;
6. the strain rates of concrete and steel reinforcements and the updated dynamic properties of materials;

Dynamic model 1: continuous beam

The equation of motion is a nonlinear PDE with variable coefficients. Its solution can be obtained by means of a numerical approach. An iterative procedure is performed, which consists in evaluating at each time step the following quantities:

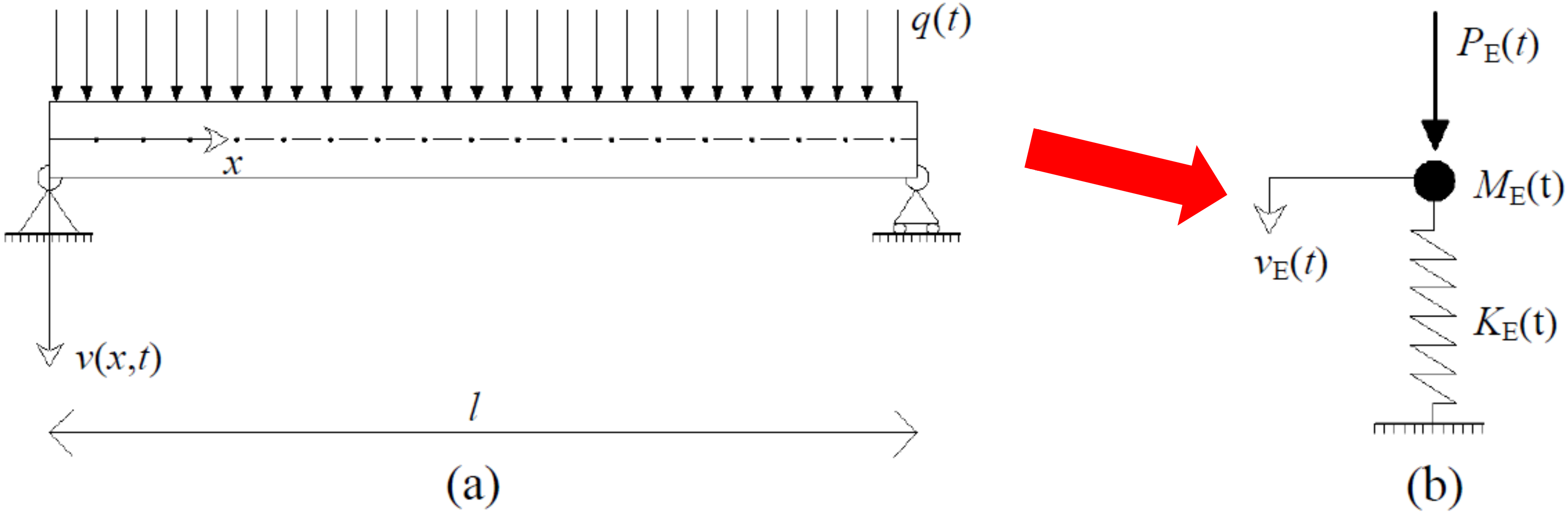
$$\bar{K}(t)\operatorname{sech}^2\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\bar{K}(t)}{\bar{M}(t)}\tanh\left(\frac{\bar{K}(t)}{\bar{M}(t)}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2+\frac{\partial^4 v(x,t)}{\partial x^4}\right]+\mu\frac{\partial^2 v(x,t)}{\partial t^2}=q(x,t)$$

1. the vertical displacement v, which is obtained by solving the equation of motion where $\bar{K}$ and $\bar{M}$ are varied at each time step due to strain rate effects;
2. the curvature $\theta = -\partial^2 v/\partial x^2$ and the rate of curvature $\partial\theta/\partial t$;
3. the bending moment M corresponding to the curvature θ;
4. the neutral axis depth from rotational equilibrium around the tensile reinforcement under the applied bending moment M;
5. the strains of concrete and steel reinforcements by using the linear deformation diagram and the value of curvature;
6. the strain rates of concrete and steel reinforcements and the updated dynamic properties of materials;
7. the updated values of the mechanical characteristics (M_y,M_u,x_y,x_u), by which the values of $\bar{K}$ and $\bar{M}$ are modified.

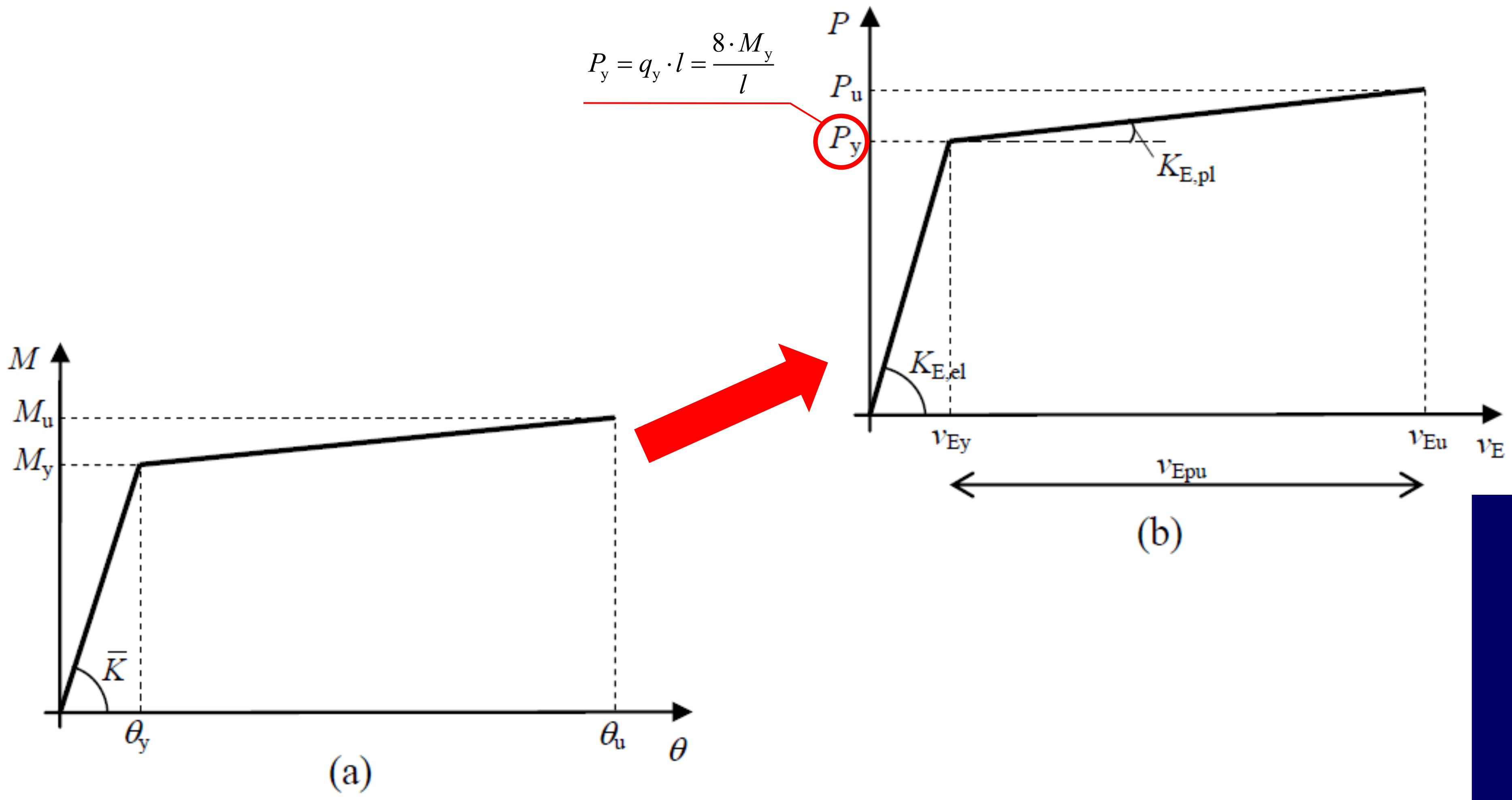
The loop is closed when the collapse criterion, which has been defined as the attainment of the maximum concrete strain (ultimate state), is satisfied.

SDOF Model

- Single Degree of Freedom Model
- Damping is disregarded, since successive cycles of loading are not considered. The first peak displacement is the more severe condition.

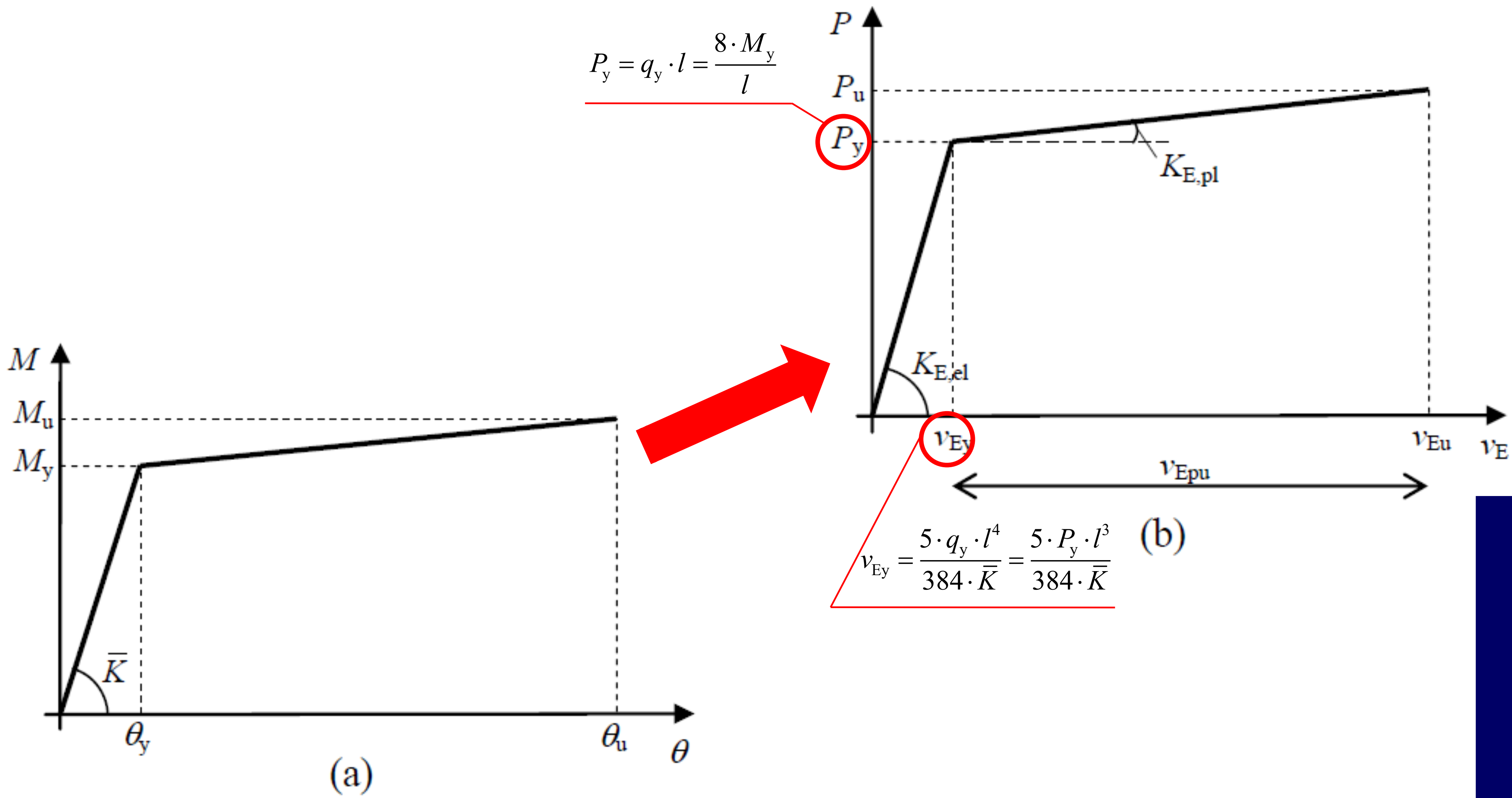


SDOF Model



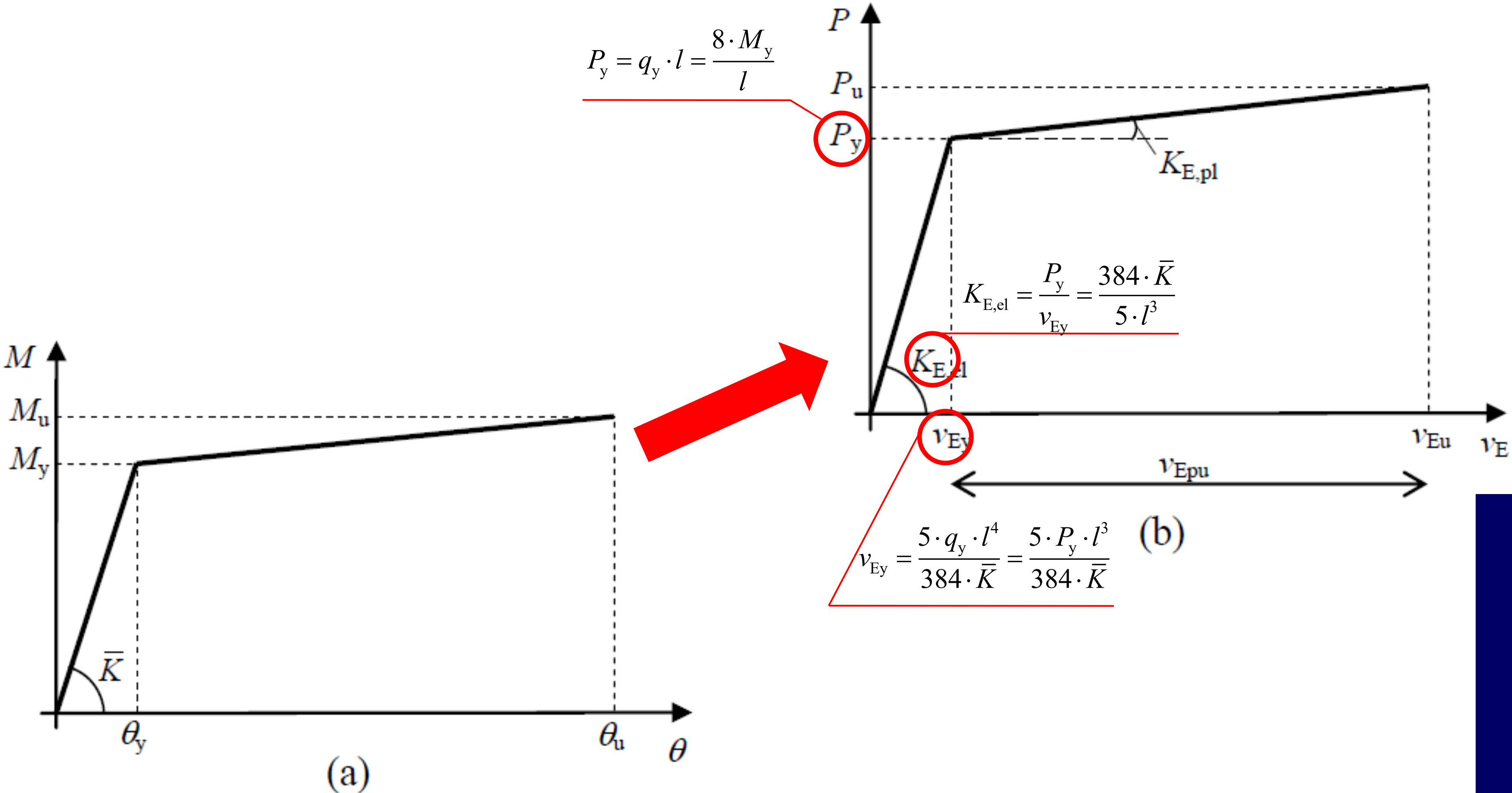
$P_y = q_y \cdot l = \frac{8 \cdot M_y}{l}$
P
P_u
P_y
$K_{E,pl}$
$K_{E,el}$
v_{Ey}
v_{Eu}
v_E
v_{Epu}
(b)
M
M_u
M_y
$\bar{K}$
θ_y
θ_u
θ
(a)

SDOF Model



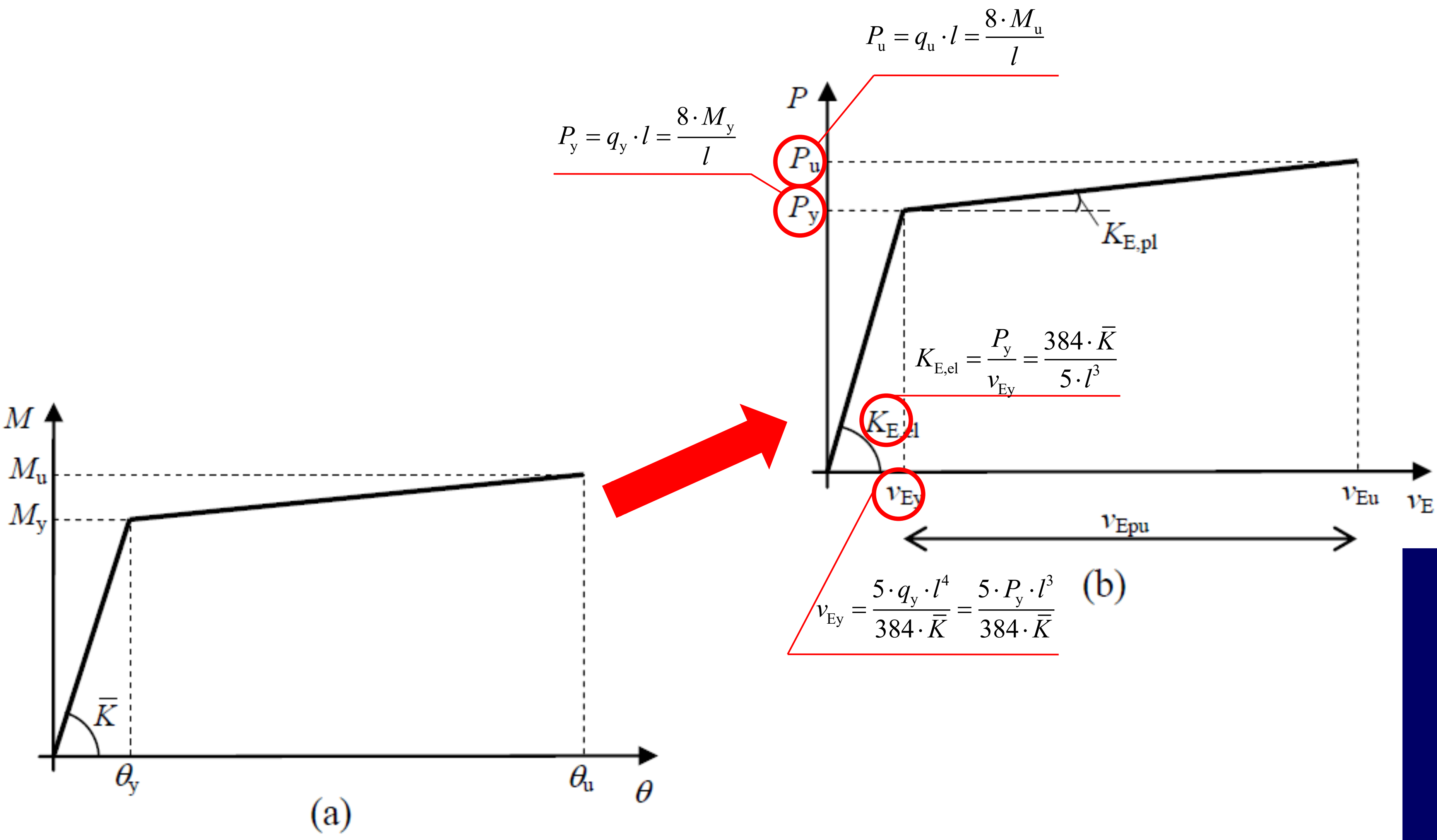
$P_y = q_y \cdot l = \frac{8 \cdot M_y}{l}$
P
P_u
P_y
$K_{E,pl}$
$K_{E,el}$
v_{Ey}
v_{Eu}
v_E
v_{Epu}
(b)
$v_{Ey} = \frac{5 \cdot q_y \cdot l^4}{384 \cdot \overline{K}} = \frac{5 \cdot P_y \cdot l^3}{384 \cdot \overline{K}}$
M
M_u
M_y
$\overline{K}$
θ_y
θ_u
θ
(a)

SDOF Model



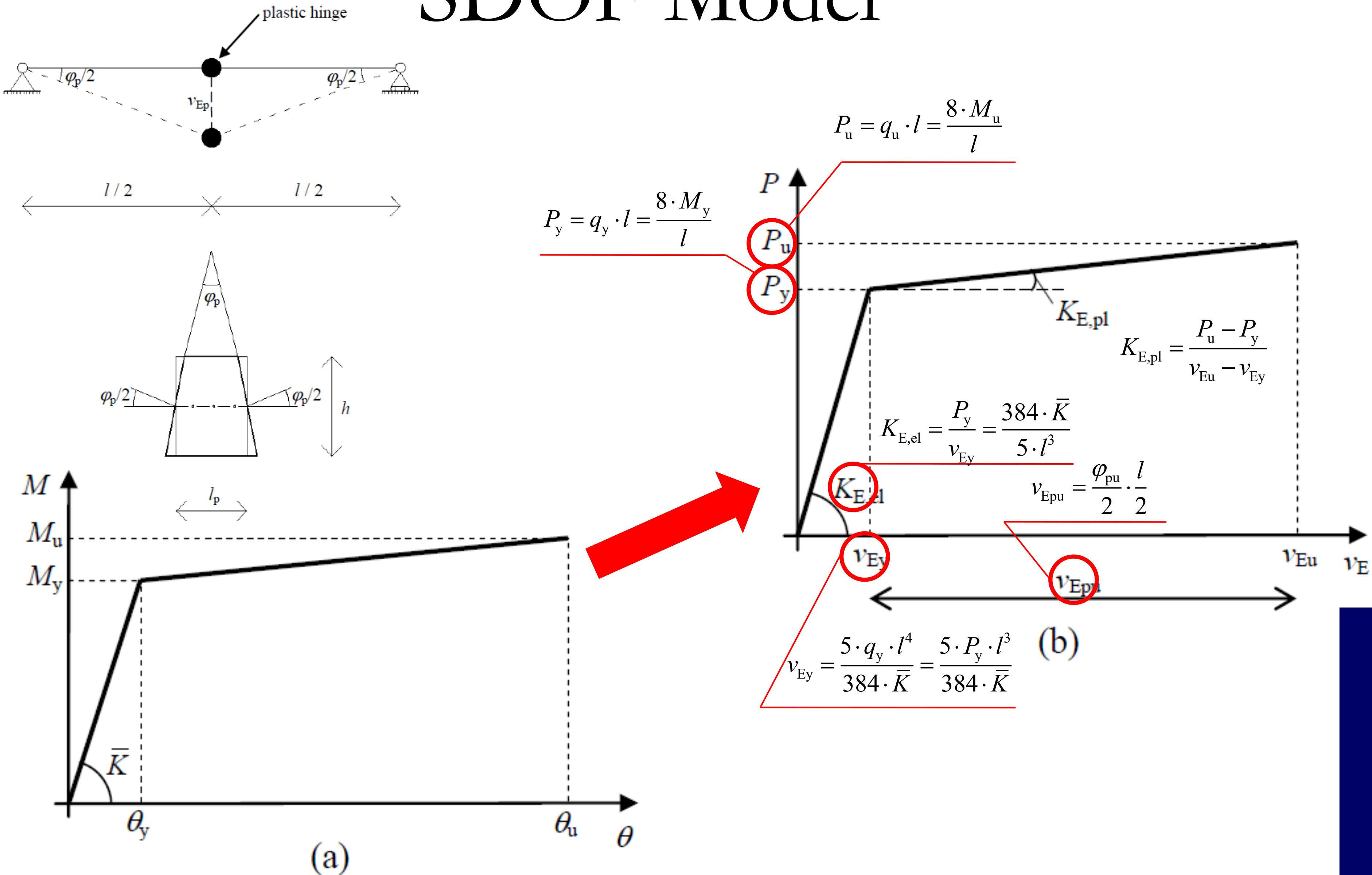
$P_y = q_y \cdot l = \frac{8 \cdot M_y}{l}$
P
P_u
P_y
$K_{E,pl}$
$K_{E,el} = \frac{P_y}{v_{Ey}} = \frac{384 \cdot \bar{K}}{5 \cdot l^3}$
$K_{E,el}$
v_{Ey}
v_{Eu}
v_E
v_{Epu}
$v_{Ey} = \frac{5 \cdot q_y \cdot l^4}{384 \cdot \bar{K}} = \frac{5 \cdot P_y \cdot l^3}{384 \cdot \bar{K}}$
(b)
M
M_u
M_y
$\bar{K}$
θ_y
θ_u
θ
(a)

SDOF Model



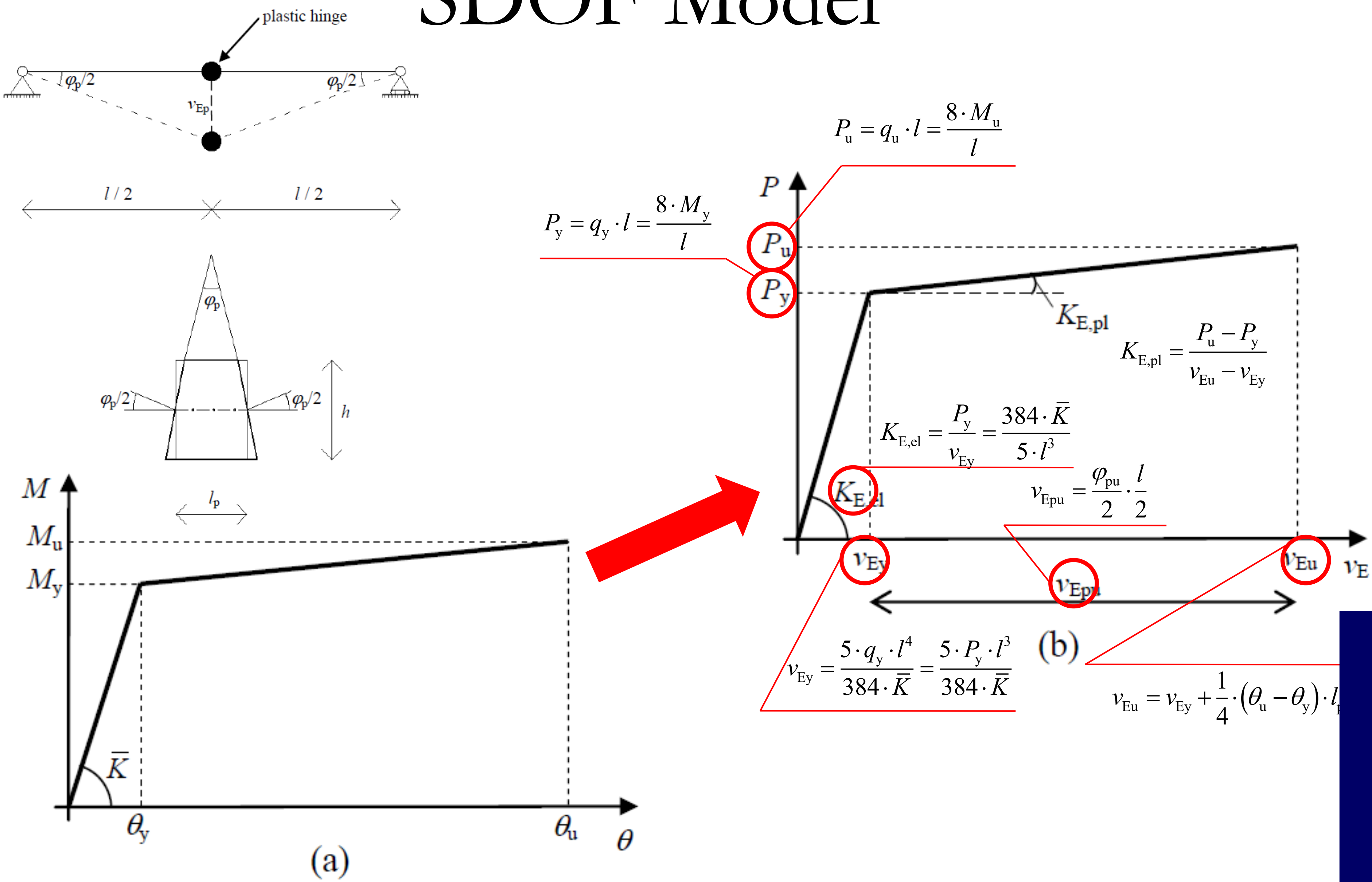
M
M_u
M_y
$\bar{K}$
θ_y
θ_u
θ
(a)
$P_u = q_u \cdot l = \frac{8 \cdot M_u}{l}$
$P_y = q_y \cdot l = \frac{8 \cdot M_y}{l}$
P
P_u
P_y
$K_{E,pl}$
$K_{E,el} = \frac{P_y}{v_{Ey}} = \frac{384 \cdot \bar{K}}{5 \cdot l^3}$
$K_{E,el}$
v_{Ey}
v_{Eu}
v_E
v_{Epu}
(b)
$v_{Ey} = \frac{5 \cdot q_y \cdot l^4}{384 \cdot \bar{K}} = \frac{5 \cdot P_y \cdot l^3}{384 \cdot \bar{K}}$

SDOF Model



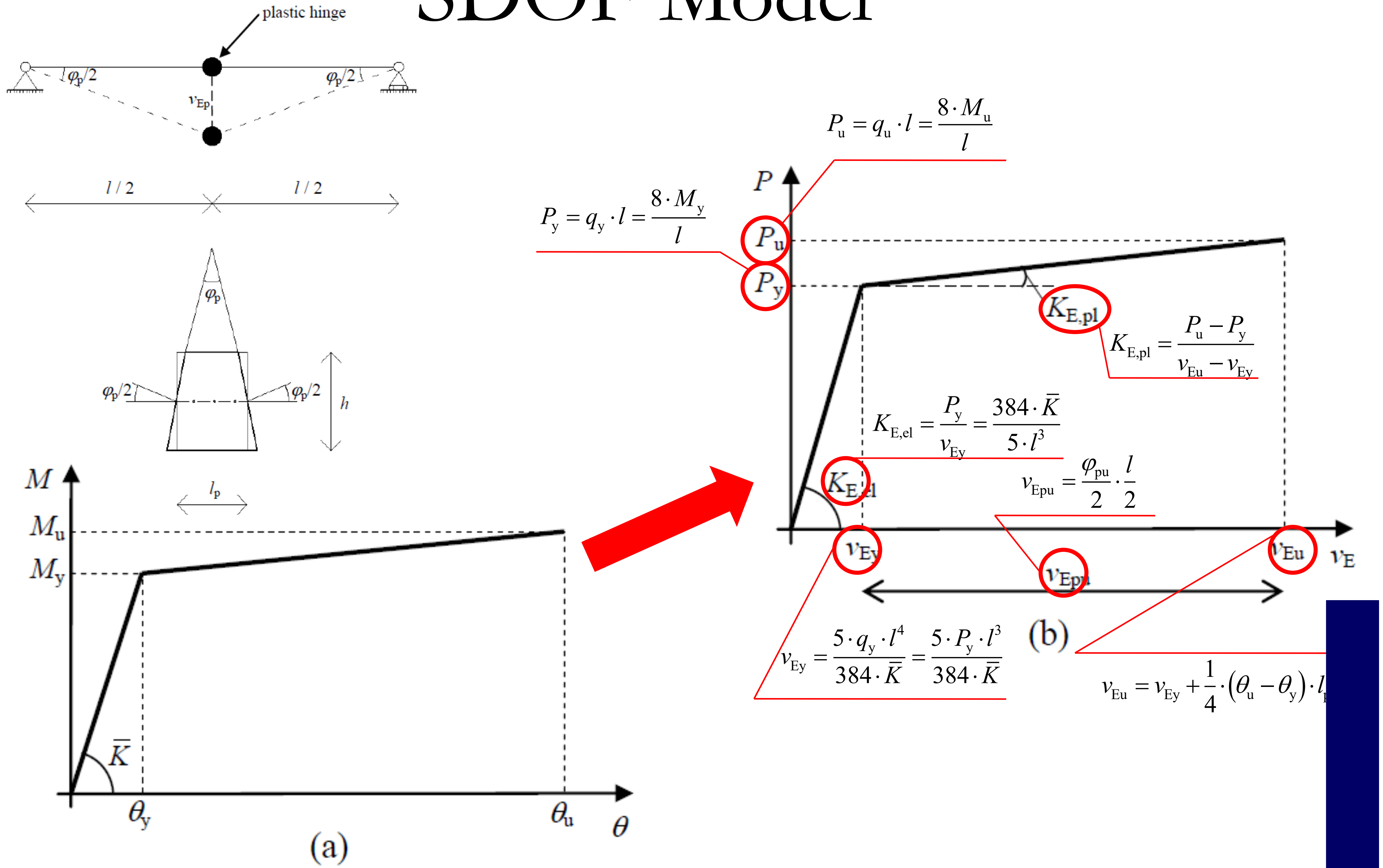
plastic hinge
$\varphi_p/2$
v_{Ep}
$l/2$
φ_p
h
l_p
M
M_u
M_y
$\bar{K}$
θ_y
θ_u
θ
(a)
$P_u = q_u \cdot l = \frac{8 \cdot M_u}{l}$
$P_y = q_y \cdot l = \frac{8 \cdot M_y}{l}$
P
P_u
P_y
$K_{E,pl}$
$K_{E,pl} = \frac{P_u - P_y}{v_{Eu} - v_{Ey}}$
$K_{E,el} = \frac{P_y}{v_{Ey}} = \frac{384 \cdot \bar{K}}{5 \cdot l^3}$
$K_{E,el}$
$v_{Epu} = \frac{\varphi_{pu}}{2} \cdot \frac{l}{2}$
v_{Ey}
v_{Epu}
v_{Eu}
v_E
$v_{Ey} = \frac{5 \cdot q_y \cdot l^4}{384 \cdot \bar{K}} = \frac{5 \cdot P_y \cdot l^3}{384 \cdot \bar{K}}$
(b)

SDOF Model



plastic hinge
$\varphi_p/2$
v_{Ep}
$l/2$
$l/2$
φ_p
h
l_p
M
M_u
M_y
$\bar{K}$
θ_y
θ_u
θ
(a)
$P_u = q_u \cdot l = \frac{8 \cdot M_u}{l}$
$P_y = q_y \cdot l = \frac{8 \cdot M_y}{l}$
P
$K_{E,pl}$
$K_{E,pl} = \frac{P_u - P_y}{v_{Eu} - v_{Ey}}$
$K_{E,el} = \frac{P_y}{v_{Ey}} = \frac{384 \cdot \bar{K}}{5 \cdot l^3}$
$K_{E,el}$
$v_{Epu} = \frac{\varphi_{pu}}{2} \cdot \frac{l}{2}$
v_{Ey}
v_{Epu}
v_{Eu}
v_E
(b)
$v_{Ey} = \frac{5 \cdot q_y \cdot l^4}{384 \cdot \bar{K}} = \frac{5 \cdot P_y \cdot l^3}{384 \cdot \bar{K}}$
$v_{Eu} = v_{Ey} + \frac{1}{4} \cdot (\theta_u - \theta_y) \cdot l$

SDOF Model



plastic hinge
$\varphi_p/2$
v_{Ep}
$l/2$
φ_p
h
l_p
M
M_u
M_y
$\bar{K}$
θ_y
θ_u
θ
(a)
$P_u = q_u \cdot l = \frac{8 \cdot M_u}{l}$
$P_y = q_y \cdot l = \frac{8 \cdot M_y}{l}$
P
P_u
P_y
$K_{E,pl}$
$K_{E,pl} = \frac{P_u - P_y}{v_{Eu} - v_{Ey}}$
$K_{E,el} = \frac{P_y}{v_{Ey}} = \frac{384 \cdot \bar{K}}{5 \cdot l^3}$
$K_{E,el}$
$v_{Epu} = \frac{\varphi_{pu}}{2} \cdot \frac{l}{2}$
v_{Ey}
v_{Epu}
v_{Eu}
v_E
(b)
$v_{Ey} = \frac{5 \cdot q_y \cdot l^4}{384 \cdot \bar{K}} = \frac{5 \cdot P_y \cdot l^3}{384 \cdot \bar{K}}$
$v_{Eu} = v_{Ey} + \frac{1}{4} \cdot (\theta_u - \theta_y) \cdot l$

SDOF Model

Elastic Case

Plastic Case

$$\theta_E = \frac{48 \cdot v_E}{5 \cdot l^2}$$

$$\theta_E = \theta_y + \frac{\varphi_p}{l_p} = \theta_y + 2 \cdot \frac{v_E - v_{Ey}}{l/2} \cdot \frac{1}{l_p}$$

$$\dot{\theta}_E = \frac{48 \cdot \dot{v}_E}{5 \cdot l^2}$$

$$\dot{\theta}_E = 2 \cdot \frac{\dot{v}_E}{l/2} \cdot \frac{1}{l_p}$$

$$M = f(\theta)$$

$$\rightarrow x \rightarrow \varepsilon$$

SDOF Model

Taking into account strain rate effect the equation of motion become a nonlinear differential equation with variable coefficients:

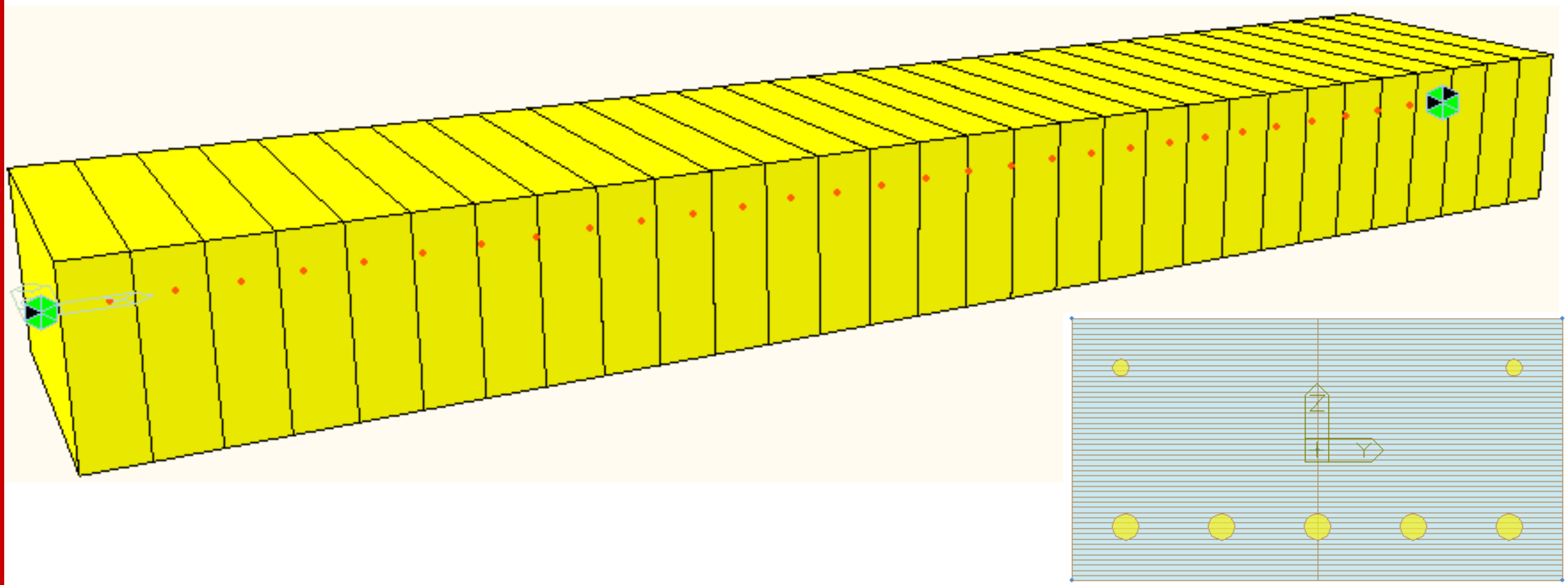
$$M_{\mathrm{E,el}} \frac{\mathrm{d}^2 v_{\mathrm{E}}(t)}{\mathrm{d}t^2} + K_{\mathrm{E,el}}(t)\, v_{\mathrm{E}}(t) = P_{\mathrm{E}}(t) \quad \text{for } 0 \le v_{\mathrm{E}} \le v_{\mathrm{Ey}}$$

$$M_{\mathrm{E,pl}} \frac{\mathrm{d}^2 v_{\mathrm{E}}(t)}{\mathrm{d}t^2} + K_{\mathrm{E,pl}}(t)\, v_{\mathrm{E}}(t) + \left(K_{\mathrm{E,el}}(t) - K_{\mathrm{E,pl}}(t)\right) v_{\mathrm{Ey}} = P_{\mathrm{E}}(t) \quad \text{for } v_{\mathrm{Ey}} < v_{\mathrm{E}} \le v_{\mathrm{Eu}}$$

$$\boxed{\begin{aligned} P_{\mathrm{E}}(t) &= q \cdot l \\ M_{\mathrm{E,pl}} &= 0.66 \cdot M_b \\ M_{\mathrm{E,el}} &= 0.78 \cdot M_b \end{aligned}}$$

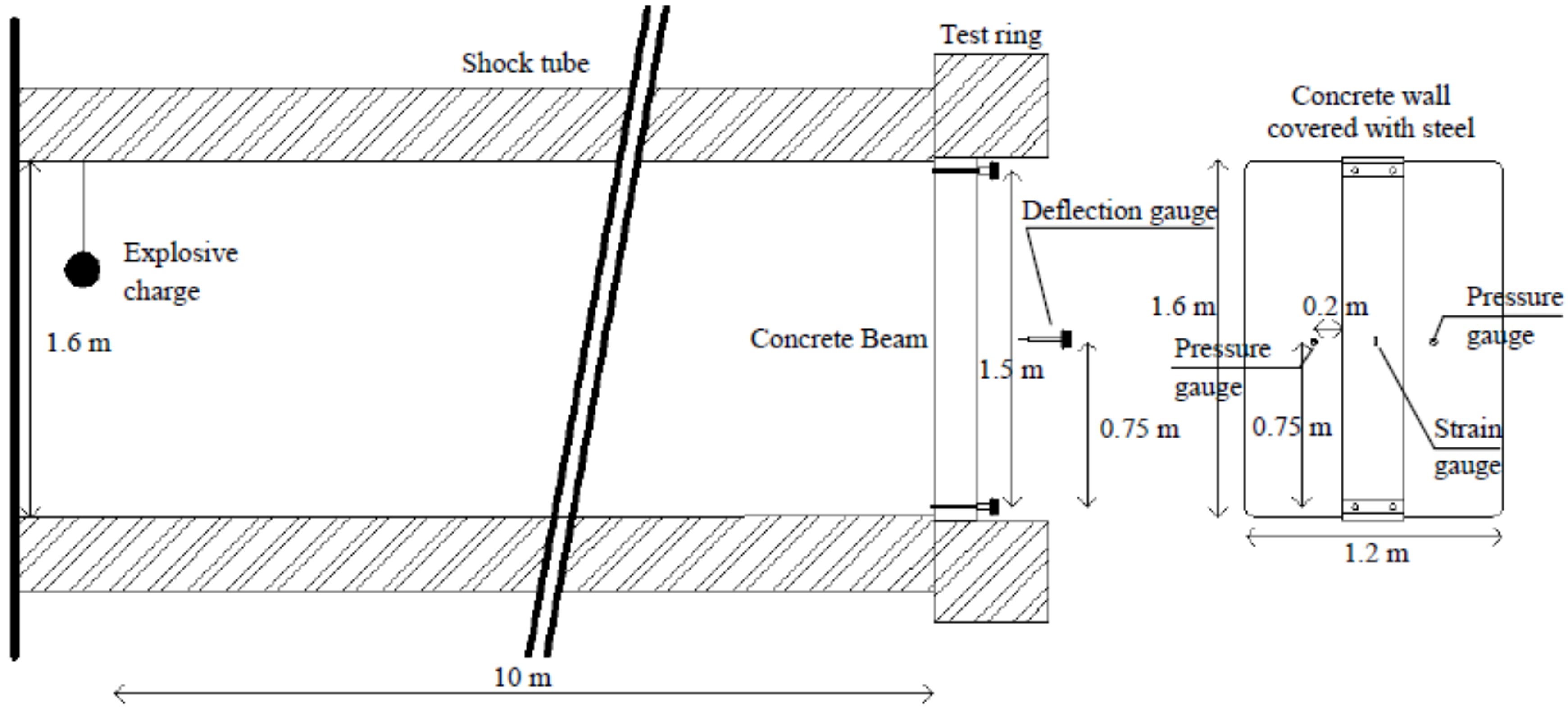
Biggs, John M., and John Melvin Biggs. Introduction to structural dynamics. McGraw-Hill College, 1964.

FEM



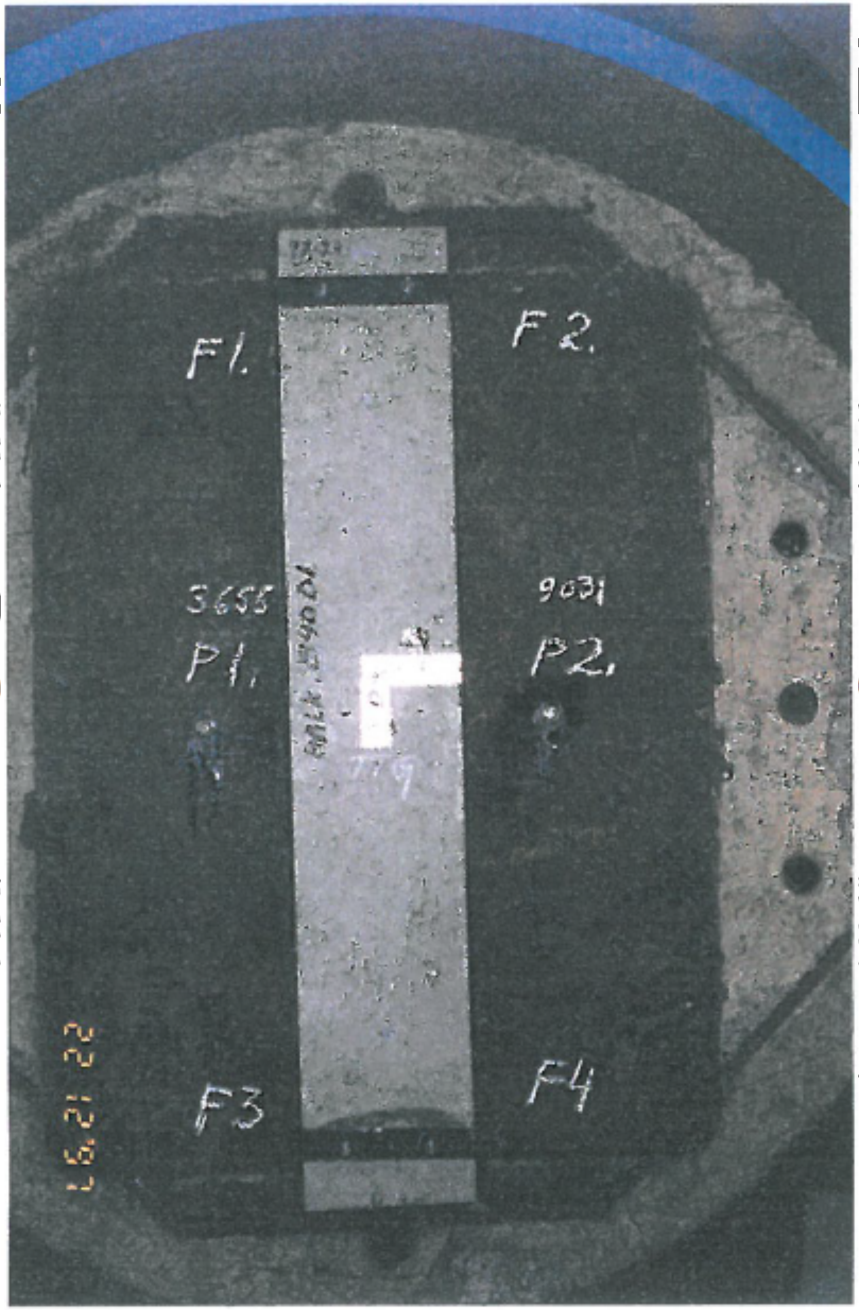
Finite element model by means of by the commercial software Midas Gen. In particular, the fiber model is used, which consists in dividing the cross-section of the beam into concrete fibers and steel rebars.

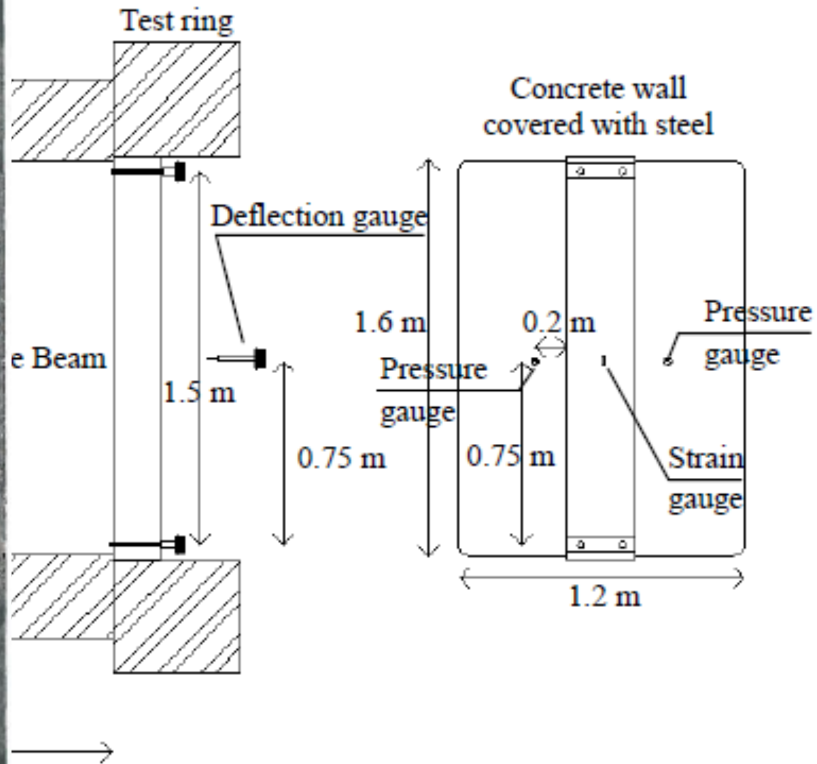
Case Study 1: Experimental Set Up



Magnusson J, Hallgren M. High performance concrete beams subjected to shock waves from air blast. Report n. FOA-R--00-01586-311--SE, Defence Research Establishment (FOA), Tumba, Sweden; 2000.

C imental Set Up



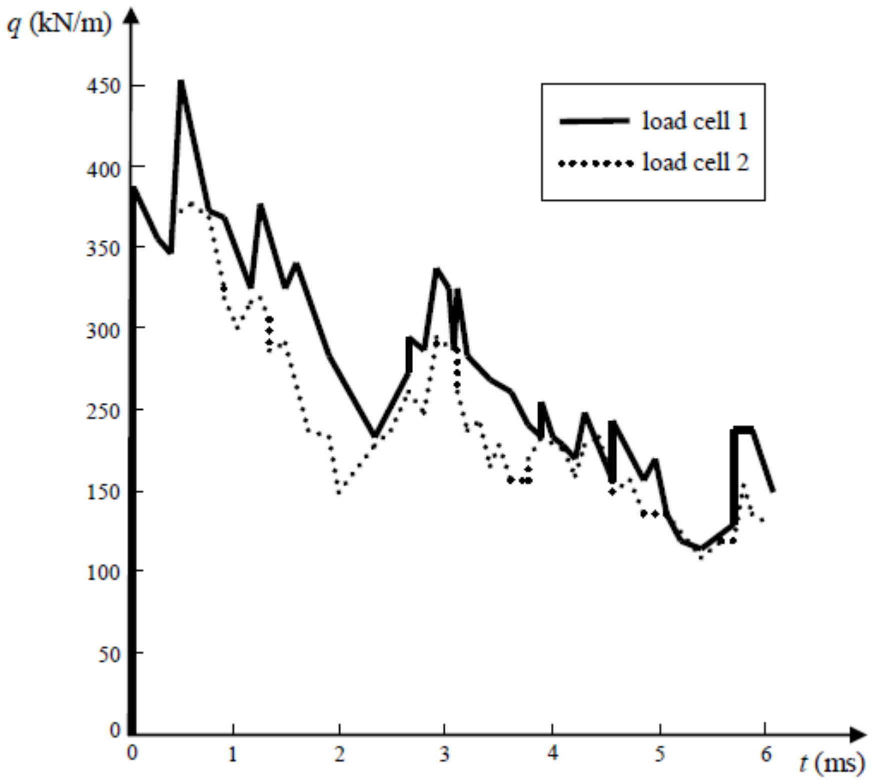


Case Study 1: Beam Characteristics

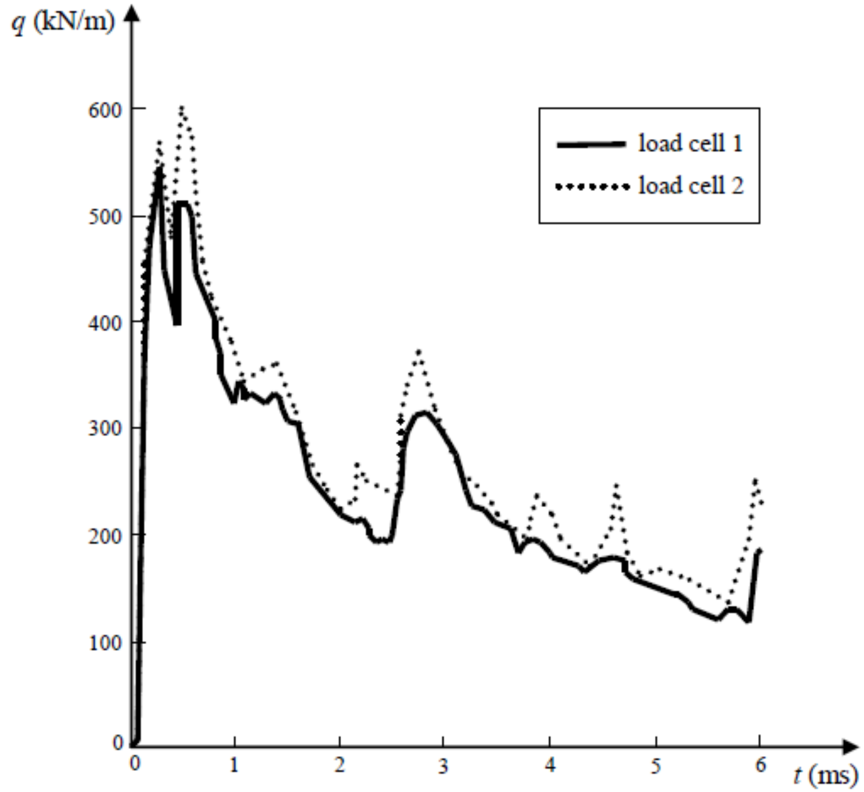
Beam label	**B40_D5**	**B200/40_D3**
Span length	1.5 m	1.5 m
Width of cross-section	0.300 m	0.293 m
Depth of cross-section	0.160 m	0.160 m
Cover	0.025 m	0.025 m
Tensile reinforcement	5 ϕ 16 mm	5 ϕ 16 mm
Compressive reinforcement	2 ϕ 10 mm	2 ϕ 10 mm
Concrete compressive strength[a]	43 MPa	173/54 MPa [b]
Maximum concrete strain registered	3.69‰	5.03‰
Steel yield strength	604 MPa	555 MPa
Steel elastic modulus	210 GPa	204 GPa
Mass per unit length	--- [c]	130 kg/m

[a] Referring to the compressive strength of ϕ150x300 mm concrete cylinders.
[b] The beam was made of two concrete layers: the first value refers to the concrete in the compressive zone, while the second is relative to the concrete in the tensile zone.
[c] This value has not been provided by the authors, so it has been assumed to be equal to120 kg/m.

Case Study 1: Recorded Load



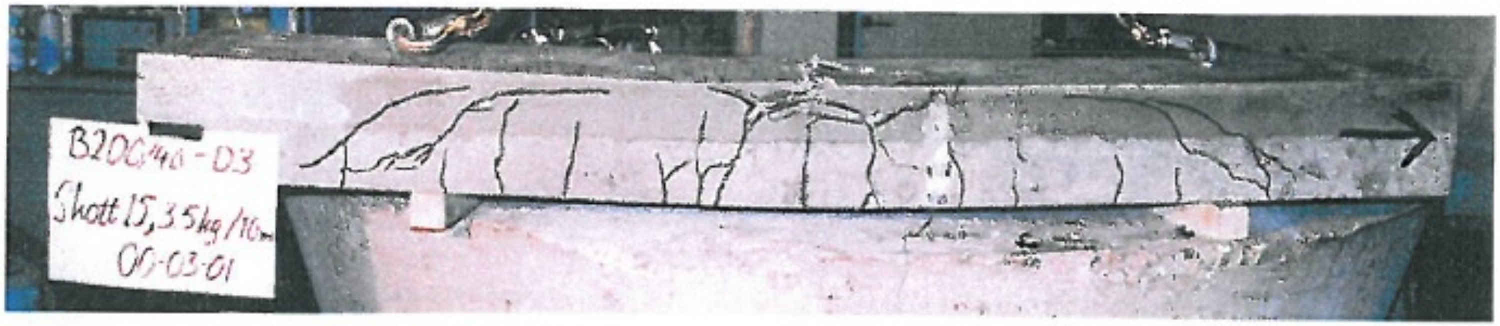
Experimental Recorded Load for beam B40_D5



Experimental Recorded Load for beam B200/40_D3

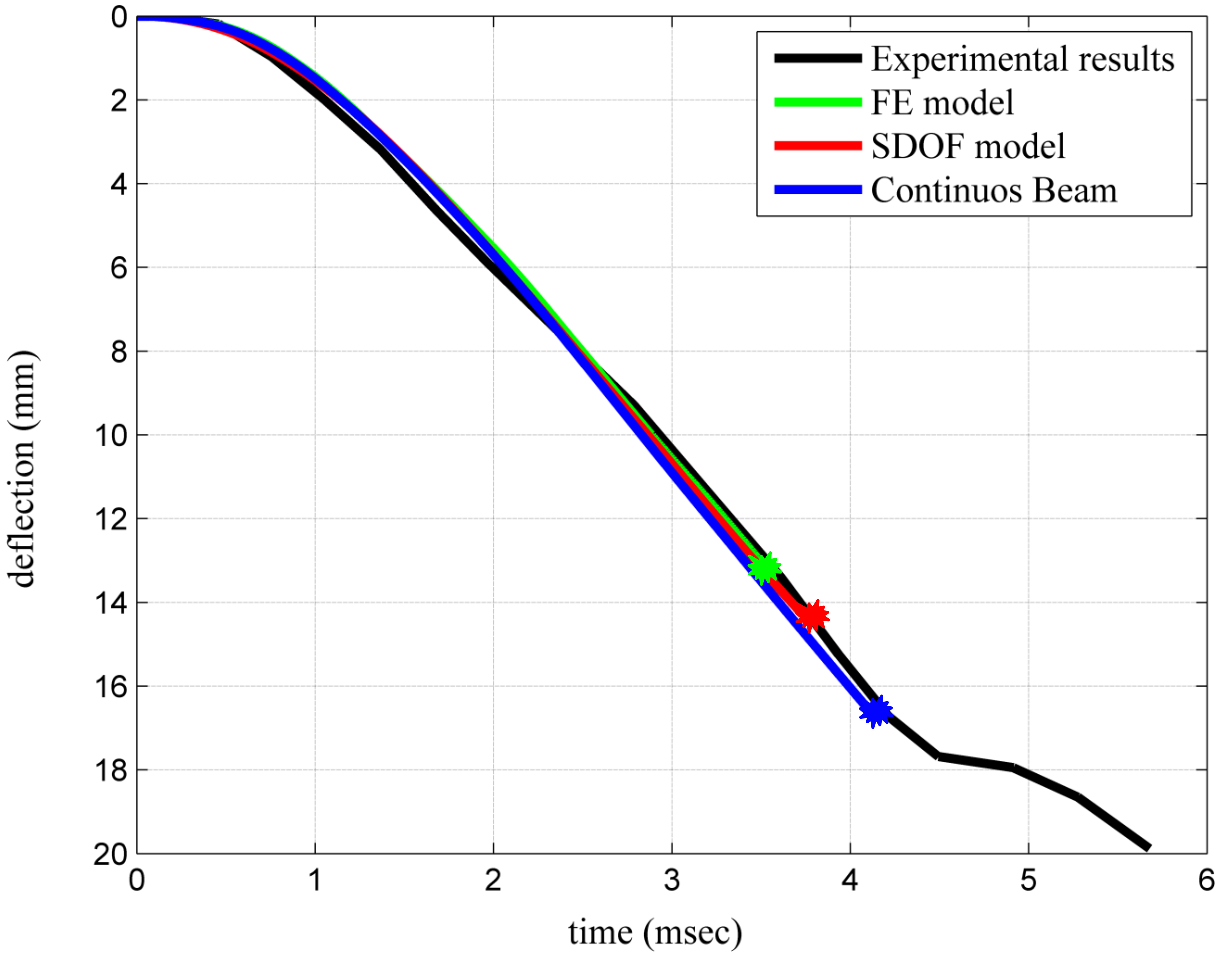
Case Study 1: Beam After Load

Beam B40_D5



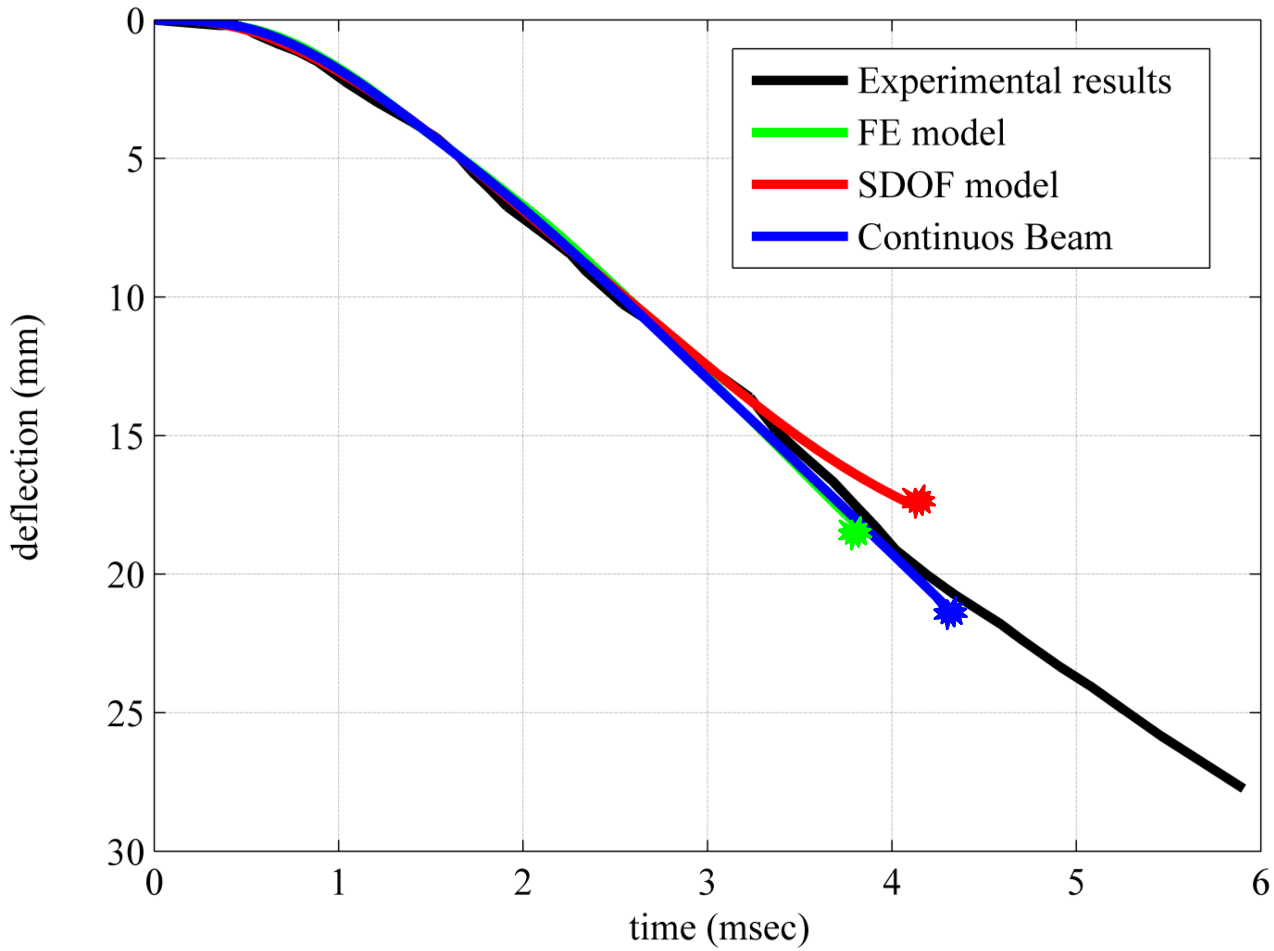
Beam B200/40_D3

Results – Case Study 1



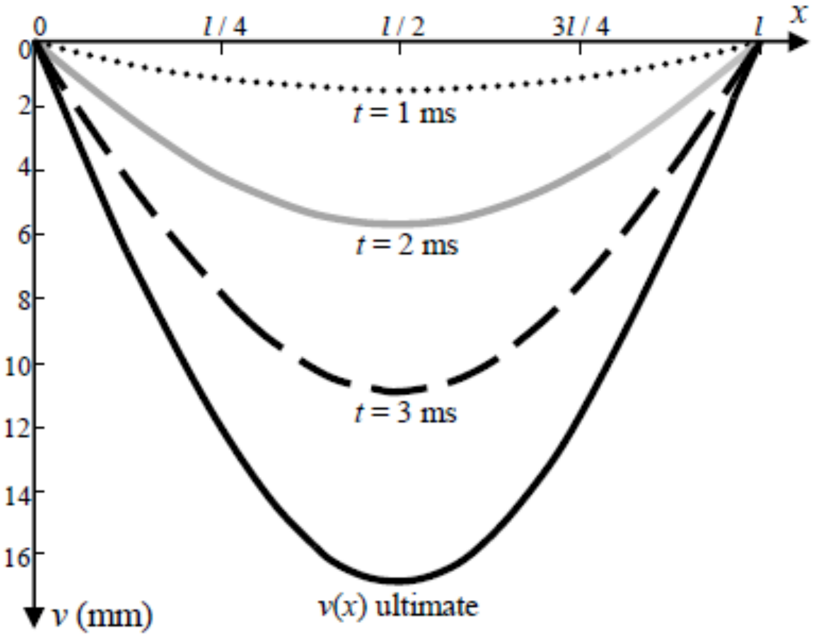
B40_D5

Results – Case Study 1

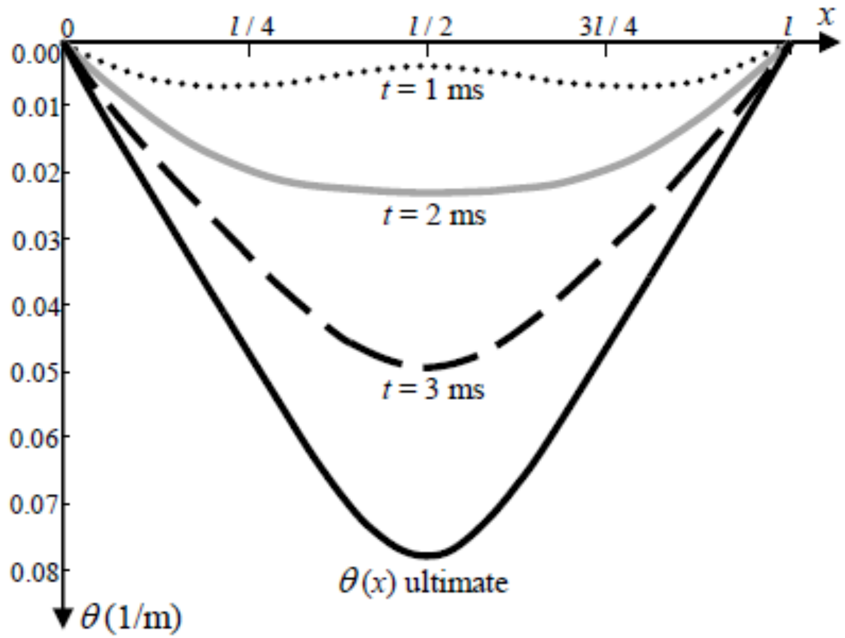


B200/40_D3

Results – Case Study 1



Displacements along longitudinal axis for beam B40_D5

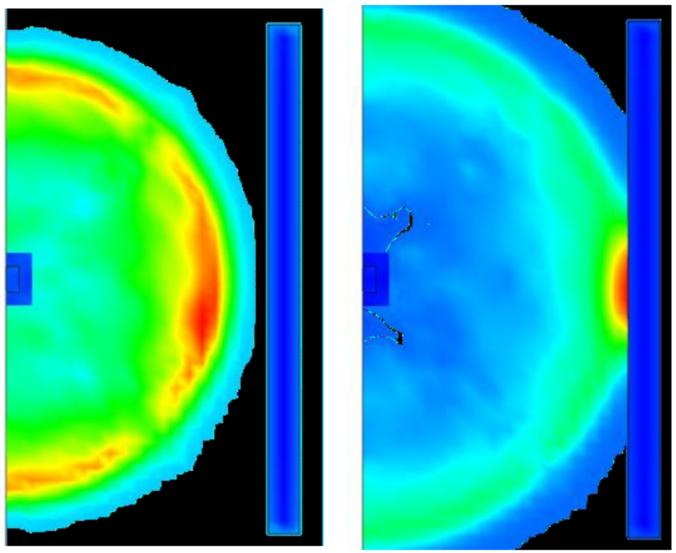


Curvatures along longitudinal axis for beam B40_D5

Outline

- **Main Aim of the Lecture**: *Simplified models for the structural behaviour of R.C. beams subjected to explosion (and impulsive) verified with experimental results. In all models, account is taken of the effects of strain-rate.*

- Section 1: Dynamic Models
- Section 2: Energy Model
- Section 3: Sensitivity Analysis
- Section 4: Tower Building Case



Energy Model

$$WE(t)$$

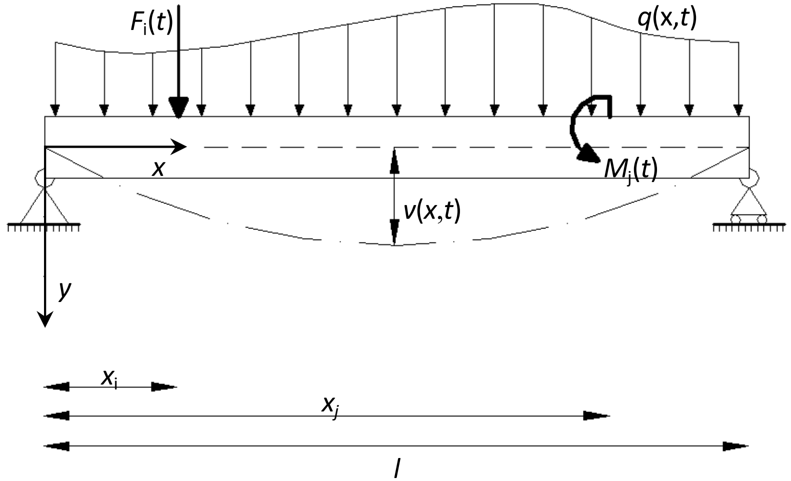
$$\int_0^l \int_0^t q(t)\cdot\frac{\partial v(x,t)}{\partial t}\,dt\,dx+\int_0^t\sum_{i=1}^n F_i(t)\frac{\partial v(x_i,t)}{\partial t}\mathrm{d}t+\int_0^t\sum_{j=1}^m M_j(t)\frac{\partial}{\partial t}\left[\frac{\partial v}{\partial x}(x_j,t)\right]\mathrm{d}t=$$

$$=\int_0^l\frac{1}{2}\mu\cdot\left(\frac{\partial v(x,t)}{\partial t}\right)^2dx+\int_0^l\int_0^{\theta(x,t)}\overline{M}\tanh\left(\frac{\overline{K}}{\overline{\overline{M}}}\theta(x,t)\right)d\theta\,dx$$

$$KE(t) \qquad\qquad SE(t)$$

$$KE(t)+SE(t)$$

$$WE(t)$$



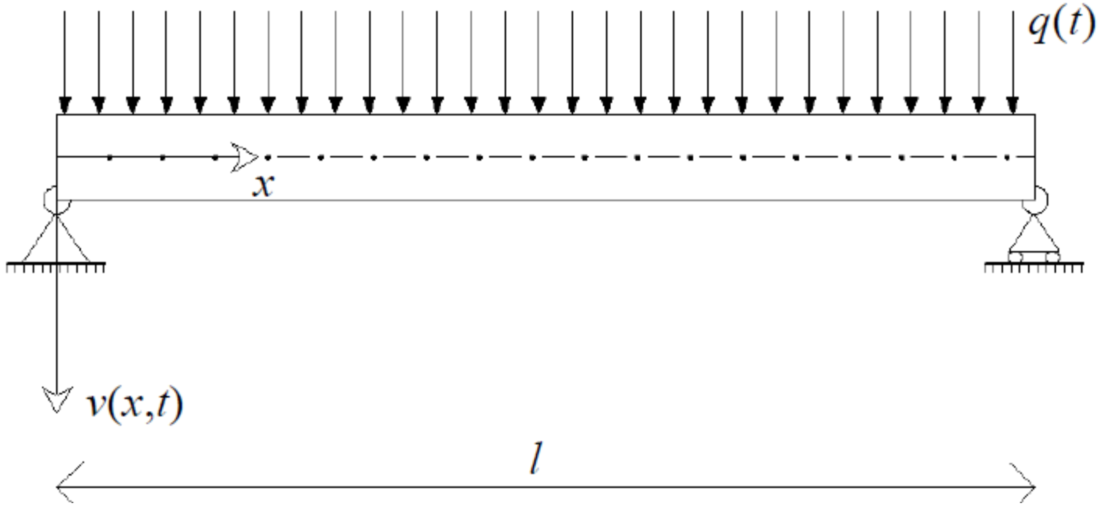
Energy Model

$$v(x,t) = V_0(t)\sin\left(\frac{\pi x}{l}\right)$$

$$\int_0^l \int_0^t q(t)\cdot\frac{\partial v(x,t)}{\partial t}dt\,dx + \int_0^t \sum_{i=1}^{n} F_i(t)\frac{\partial v(x_i,t)}{\partial t}\mathrm{d}t + \int_0^t \sum_{j=1}^{m} M_j(t)\frac{\partial}{\partial t}\left[\frac{\partial v}{\partial x}(x_j,t)\right]\mathrm{d}t =$$
$$= \int_0^l \frac{1}{2}\mu\cdot\left(\frac{\partial v(x,t)}{\partial t}\right)^2 dx + \int_0^l \int_0^{\theta(x,t)} \overline{M}\tanh\left(\frac{\overline{K}}{\overline{M}}\theta(x,t)\right)d\theta\,dx$$

$$\int_0^t \int_0^l q_0(t)\frac{\partial V_0(t)}{\partial t}\sin\left(\frac{\pi x}{l}\right)\mathrm{d}x\,\mathrm{d}t + \int_0^t \sum_{i=1}^{n} F_i(t)\frac{\partial V_0(t)}{\partial t}\sin\left(\frac{\pi x_i}{l}\right)\mathrm{d}t + \int_0^t \sum_{j=1}^{m} M_j(t)\left(\frac{\pi x_j}{l}\right)\frac{\partial V_0(t)}{\partial t}\cos\left(\frac{\pi x_j}{l}\right)\mathrm{d}t =$$
$$= \int_0^l \frac{1}{2}\mu\left[\frac{\partial V_0(t)}{\partial t}\right]^2 \sin^2\left(\frac{\pi x}{l}\right)\mathrm{d}x + \int_0^l \frac{\overline{M}^2}{\overline{K}}\ln\left\{\cosh\left[\frac{\overline{K}}{\overline{M}}\frac{\pi^2}{l^2}V_0(t)\sin\left(\frac{\pi x}{l}\right)\right]\right\}\mathrm{d}x$$

Energy Model



$$V_{0\ (j+1)}^{2}\left\{\frac{\mu l}{16k^2}\right\}+V_{0(j+1)}\left\{-\frac{\mu l}{8k^2}V_{0(j-1)}-q_{(j)}\frac{l}{\pi}\right\}+\left\{\begin{array}{l}\frac{\mu l}{16k^2}V_{0\ (j-1)}^{2}+\sum_{i=1}^{n+1}\frac{\overline{M}^2}{\overline{K}}\ln\left(\cosh\left(\left(\frac{\pi}{l}\right)^2 V_{0(j)}\sin\left(\frac{\pi x_i}{l}\right)\frac{\overline{K}}{\overline{\overline{M}}}\right)\right)h+\\ -\left(\frac{2l}{\pi}\right)\cdot\sum_{m=1}^{j-1}q_{(m)}\cdot\frac{V_{0(m+1)}-V_{0(m-1)}}{2}+q_{(j)}\frac{V_{0(j-1)}}{2}\left(\frac{l}{\pi}\right)\end{array}\right\}=0$$

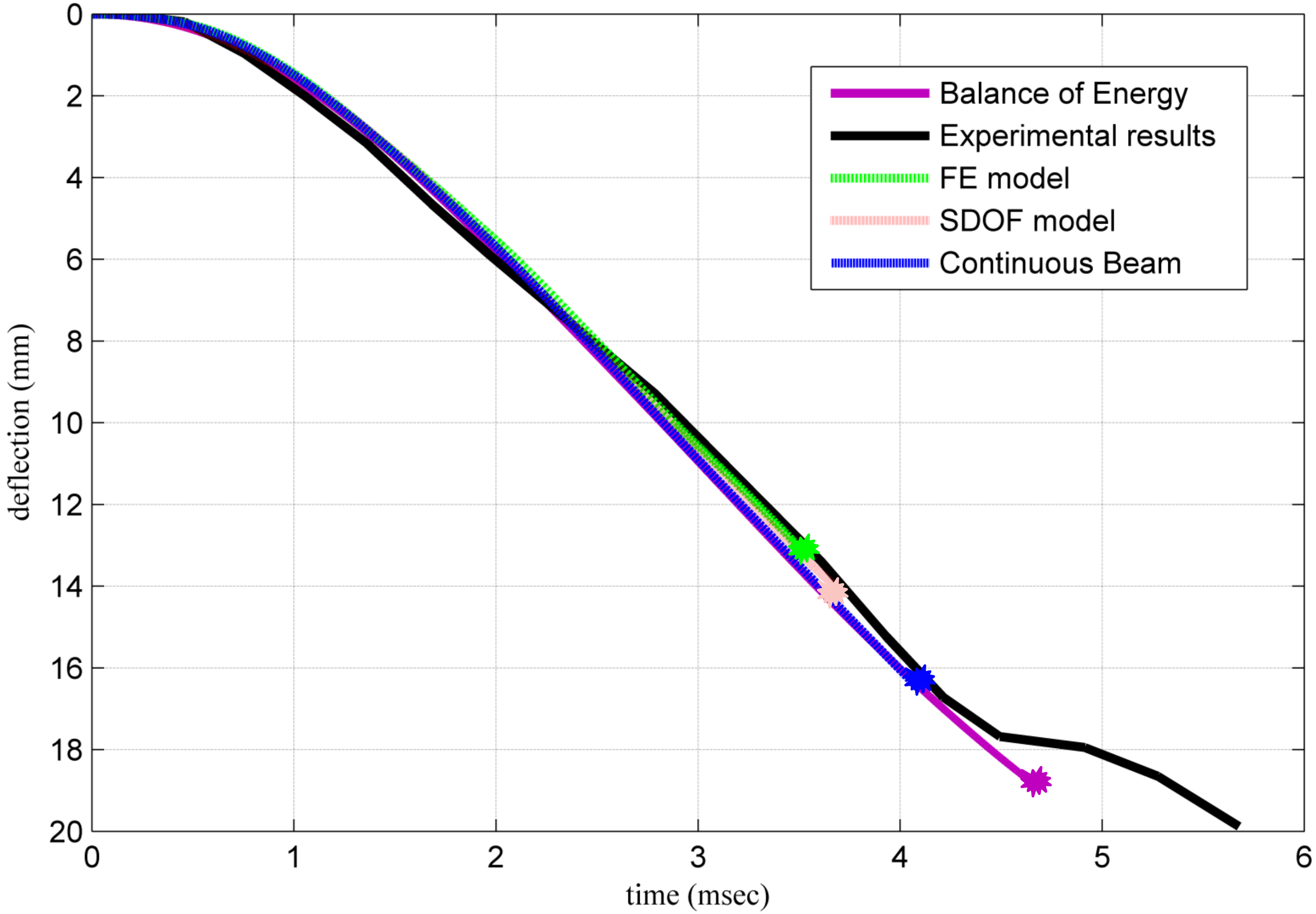
Energy Model

$$V_{0\ (j+1)}^{2}\left\{\frac{\mu l}{16k^2}\right\}+V_{0(j+1)}\left\{-\frac{\mu l}{8k^2}V_{0(j-1)}-q_{(j)}\frac{l}{\pi}\right\}+\left\{\begin{matrix}\frac{\mu l}{16k^2}V_{0\ (j-1)}^{2}+\sum_{i=1}^{n+1}\frac{\overline{M}^2}{\overline{K}}\ln\left(\cosh\left(\left(\frac{\pi}{l}\right)^2 V_{0(j)}\sin\left(\frac{\pi x_i}{l}\right)\frac{\overline{K}}{\overline{M}}\right)\right)h+\\-\left(\frac{2l}{\pi}\right)\cdot\sum_{m=1}^{j-1}q_{(m)}\cdot\frac{V_{0(m+1)}-V_{0(m-1)}}{2}+q_{(j)}\frac{V_{0(j-1)}}{2}\left(\frac{l}{\pi}\right)\end{matrix}\right\}=0$$

1. Determine the unique unknown $V_{0(j+1)}$ and calculate the sinusoidal distribution of displacements and, consequently, the curvature at midspan.
2. Then, considering previous curvature calculate the rate of curvature $= \partial\theta/\partial t$.
3. Determine the bending moment M corresponding to the curvature at time t.
4. Calculate the neutral axis depth from rotational equilibrium around the tensile reinforcement under the applied bending moment M.
5. Determine the strains of concrete and steel reinforcements by using the linear deformation diagram and the value of curvature.
6. Determine the strain rates of concrete and steel reinforcements.
7. Calculate the updated dynamic properties of materials.
8. Determine the updated values of the mechanical characteristics (χ_y, M_y, χ_u, M_u), by which the values of $\bar{K}$ and $\bar{M}$ are modified.

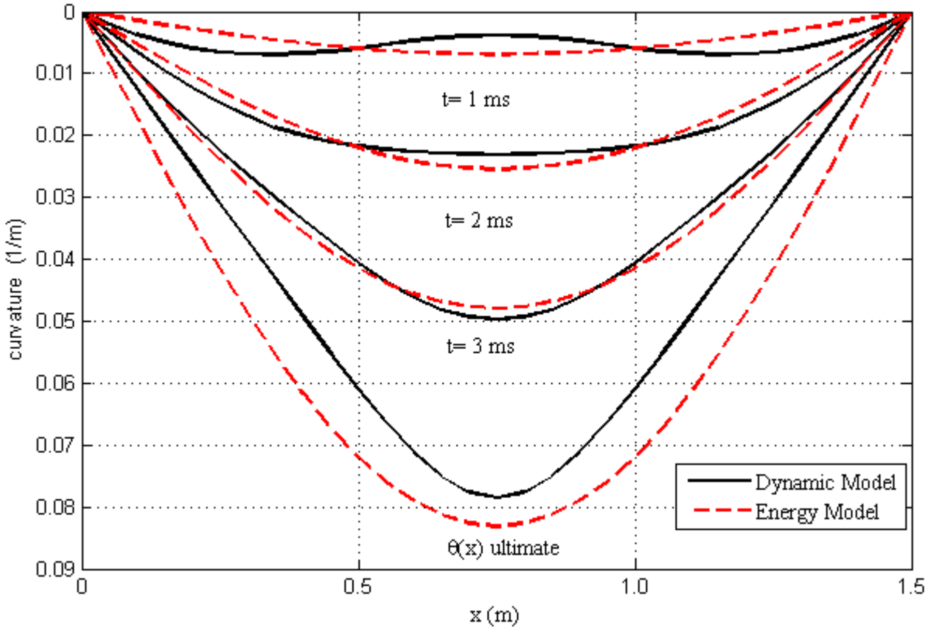
The loop is closed when the maximum concrete strain (ultimate state), is obtained.

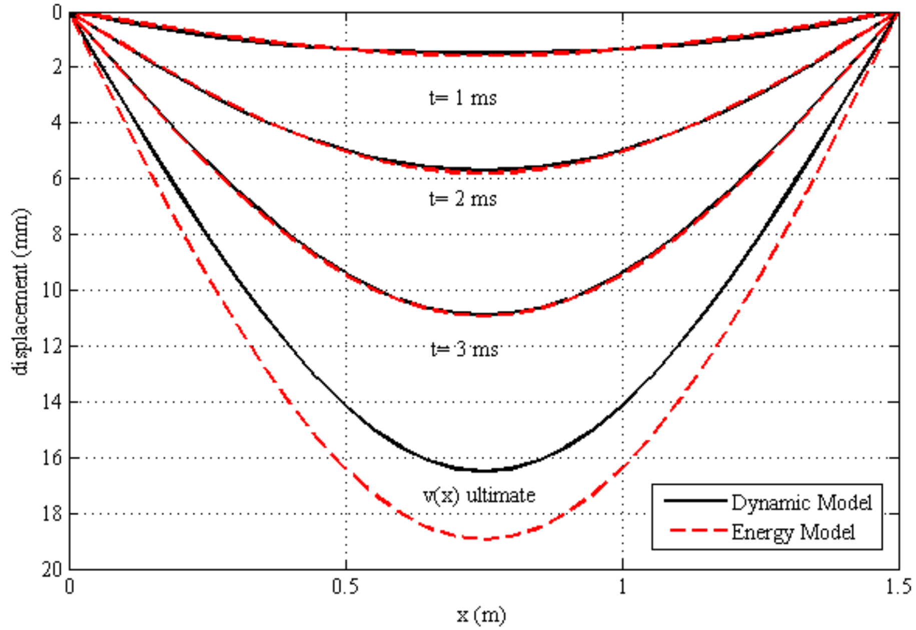
Case Study 1: Results



B40_D5

Case Study 1: Recorded Load

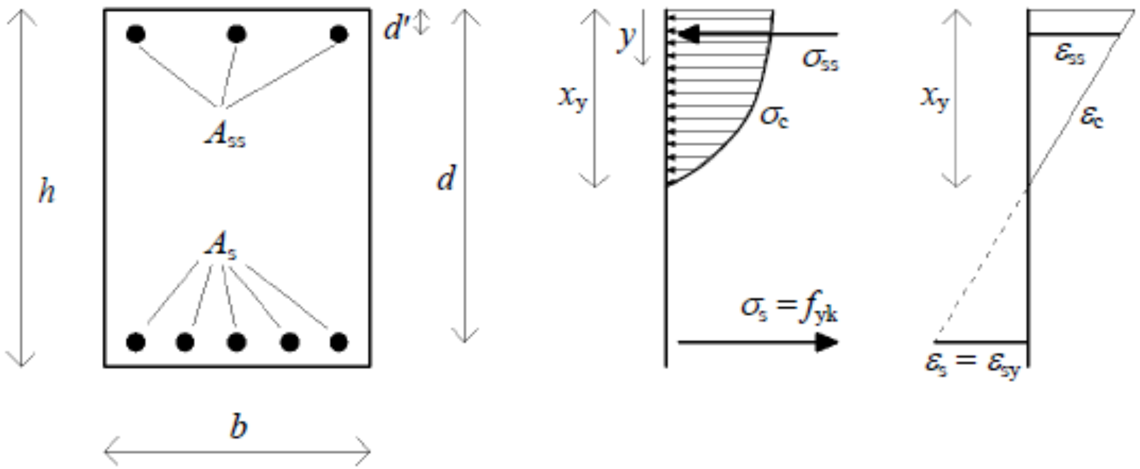




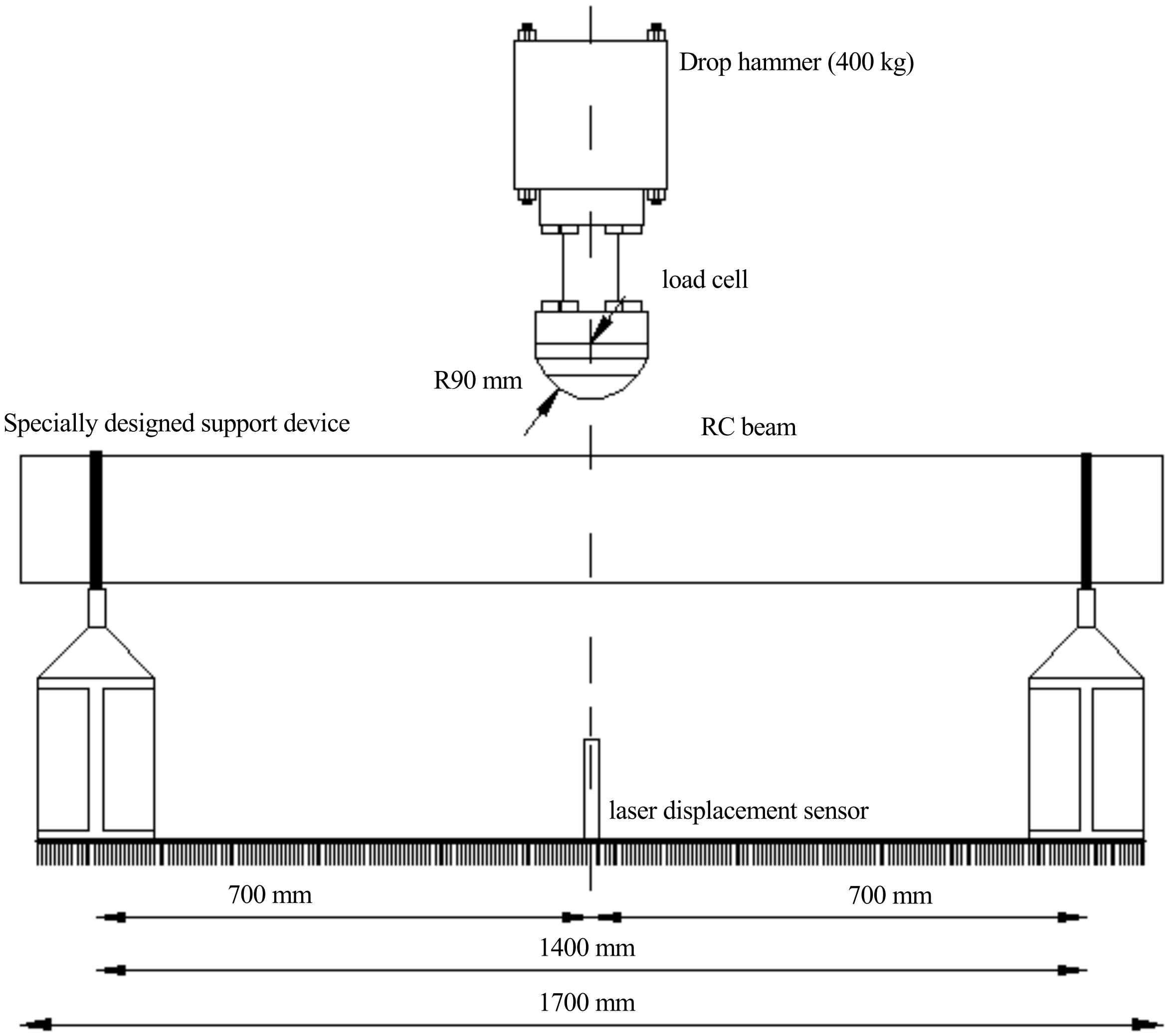
B40_D5

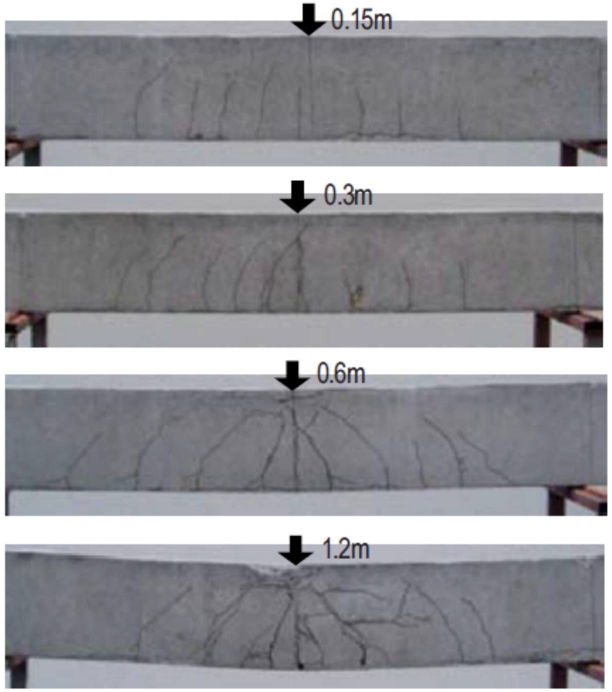
Case Study 1: Recorded Load

Beam B40_D5		**Energy**	**Continuos Beam**	**Experimental**
Max. Strain Concrete ε_c		0.0045	0.0044	0.0037
Max. Strain Tensile Reinf.	ε_s	0.0061	0.0056	
Max. Strain Compress. Reinf.	ε_{ss}	0.0020	0.0020	



Case Study 2: Experimental Set Up



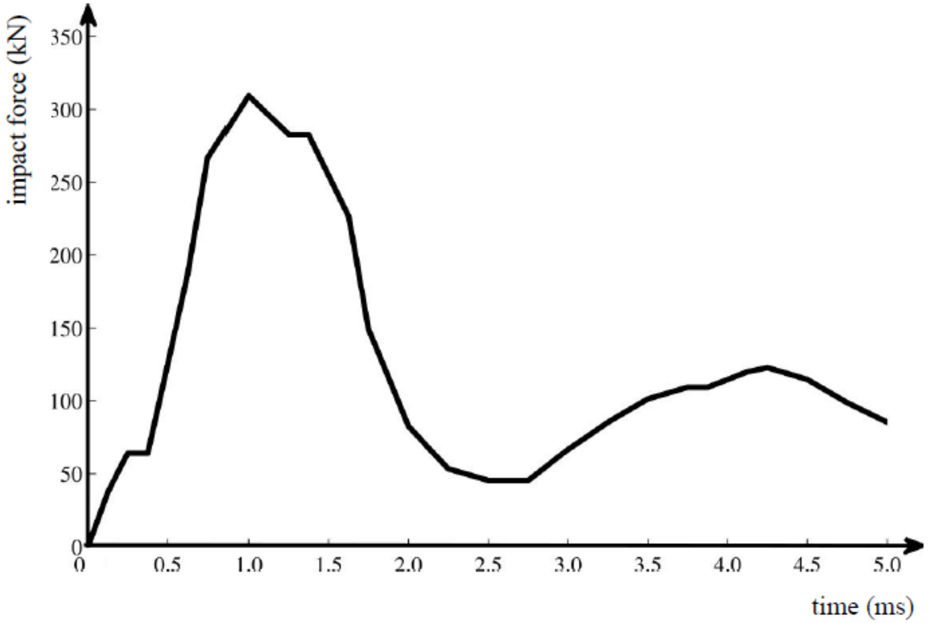


K. Fujikake, B. Li, S. Soeun, Impact response of reinforced concrete beam and its analytical evaluation, J. Struct. Eng. ASCE 135 (2009) 938-950.

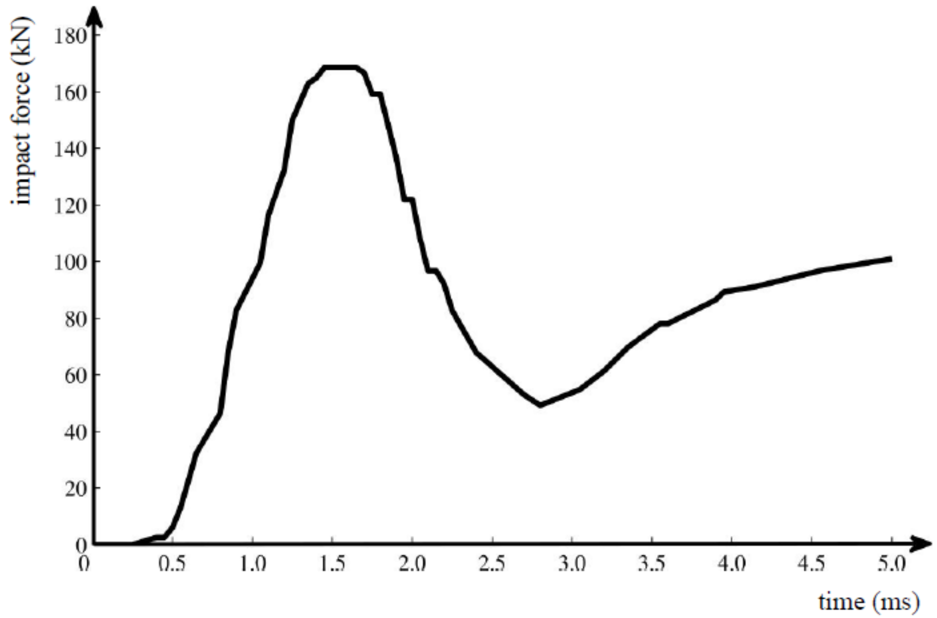
Case Study 2: Beam Characteristics

Beam label	**S1616**
Span length	1.4 m
Width of cross-section	0.150 m
Depth of cross-section	0.250 m
Cover	0.04 m
Area of tensile reinforcement	$3.97 \cdot 10^{-4}$ m^2
Area of compressive reinforcement	$3.97 \cdot 10^{-4}$ m^2
Compressive strength of concrete	42 MPa
Yield strength of reinforcing steel	426 MPa

Case Study 2: Recorded Load

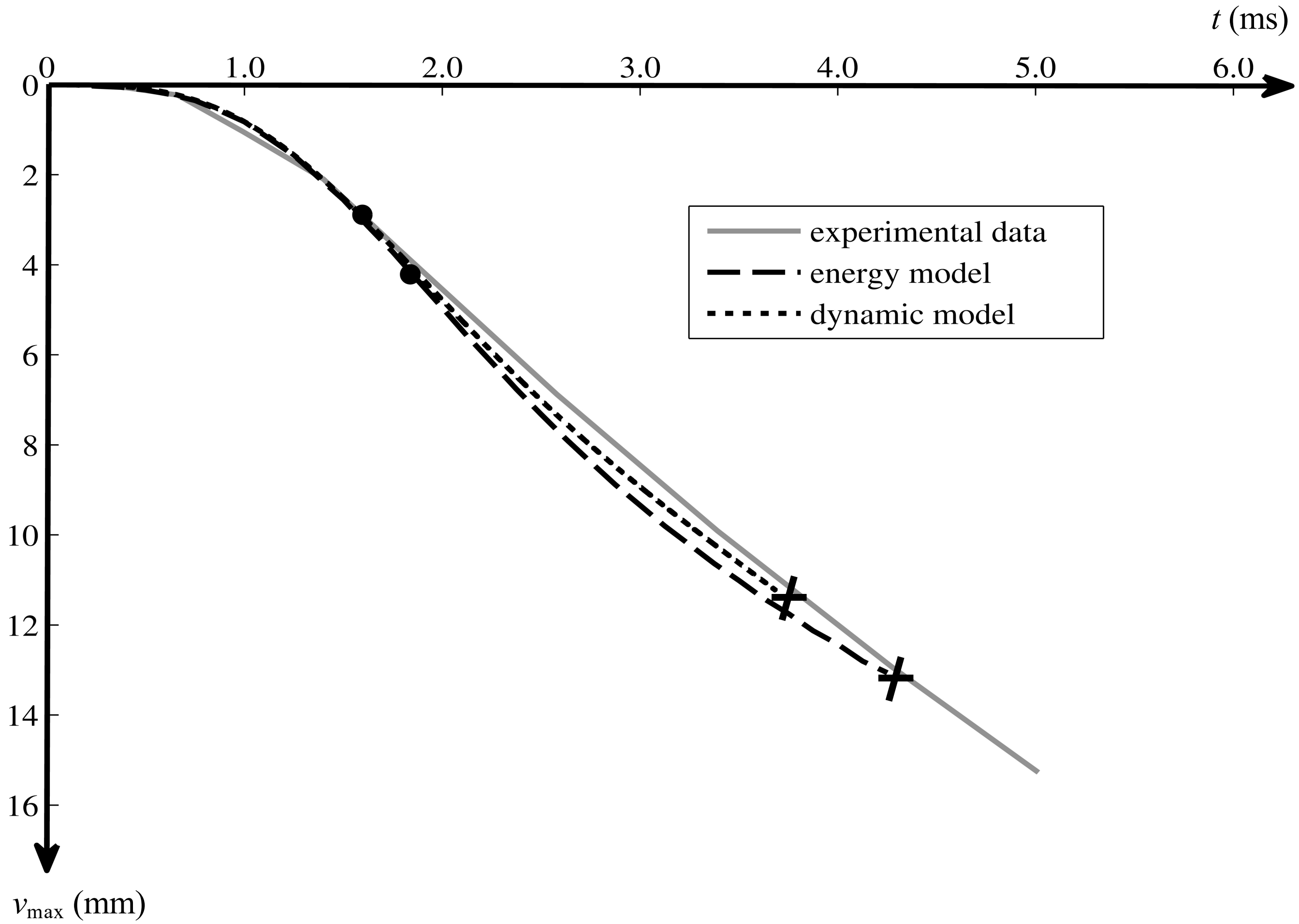


Impact force versus time for the S1616 series of beams, with a drop height equal to **1.2 m.**



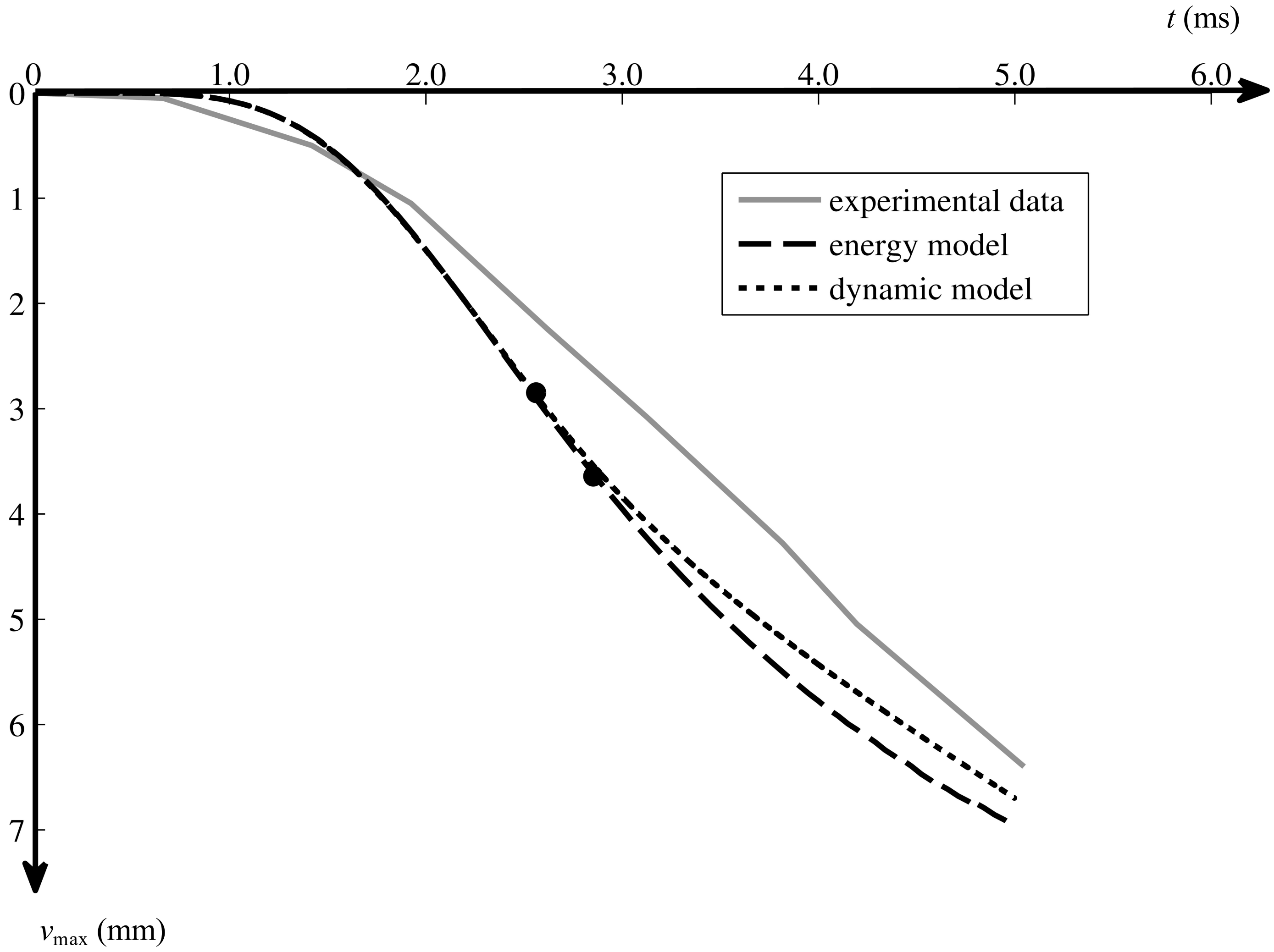
Impact force versus time for the S1616 series of beams, with a drop height equal to **0.3m.**

Case Study 2: Recorded Load



Comparison between the experimental data and the theoretical results obtained from the two models presented in this work, relative to the beam of the S1616 series subjected to the drop of a hammer from a height of 1.2 m.

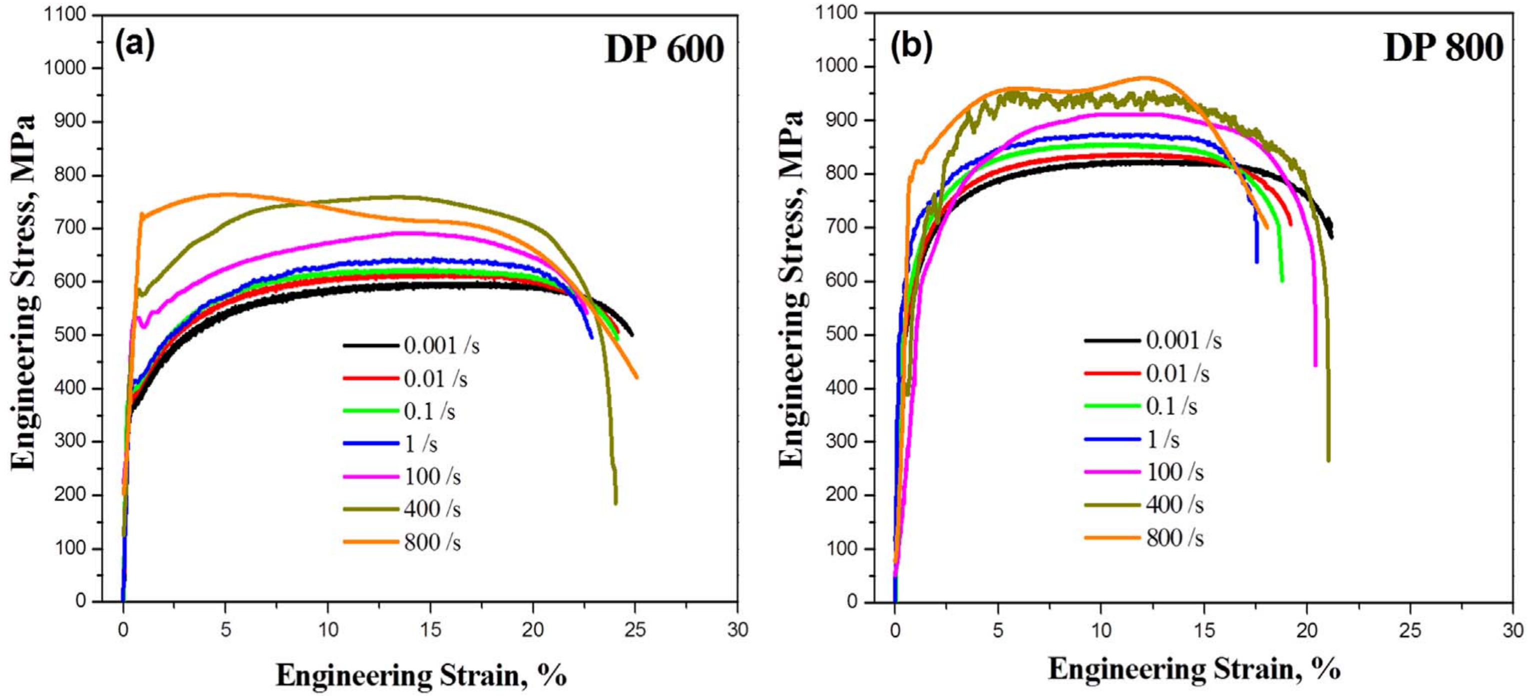
Case Study 2: Recorded Load



Comparison between the experimental data and the theoretical results obtained from the two models presented in this work, relative to the beam of the S1616 series subjected to the drop of a hammer from a height of 0.3 m.

Question:

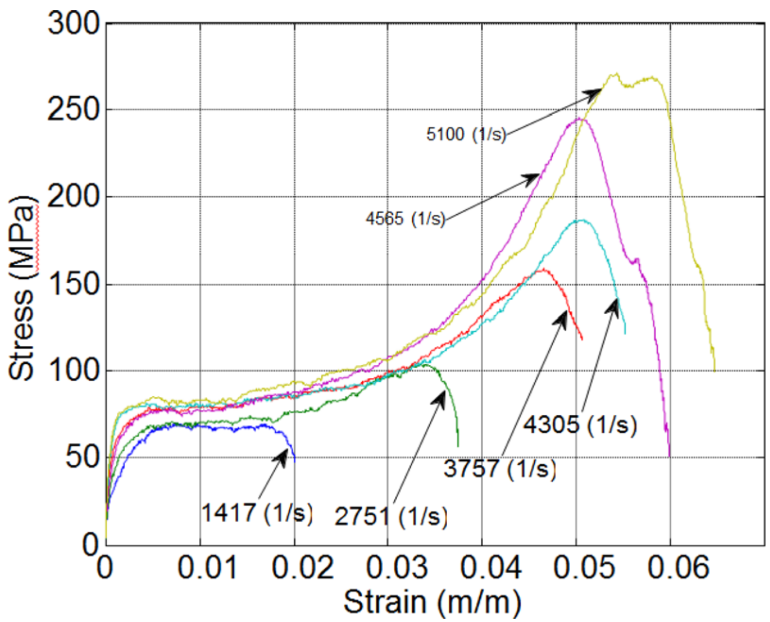
- What is the importance of Strain Rate Effects?

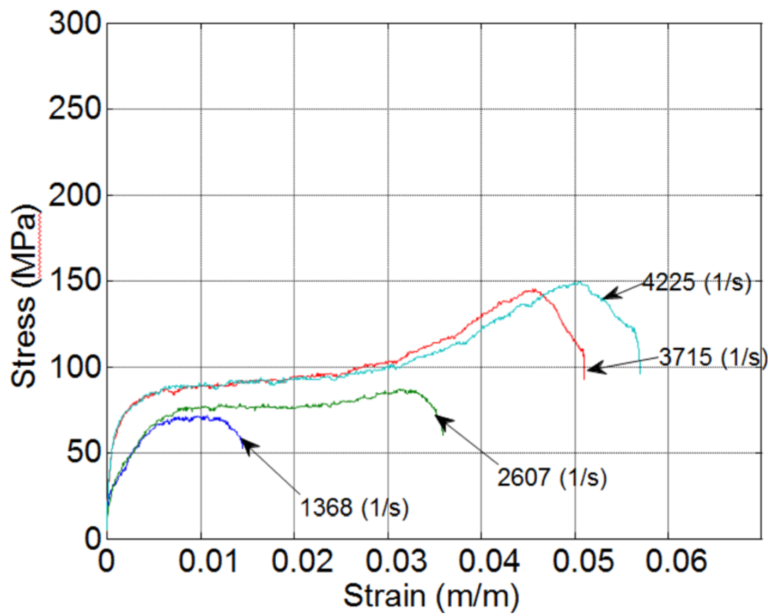


Das, Anindya, et al. "Micromechanisms of deformation in dual phase steels at high strain rates." Materials Science and Engineering: A (2016).10.1016/j.msea.2016.10.101

Question:

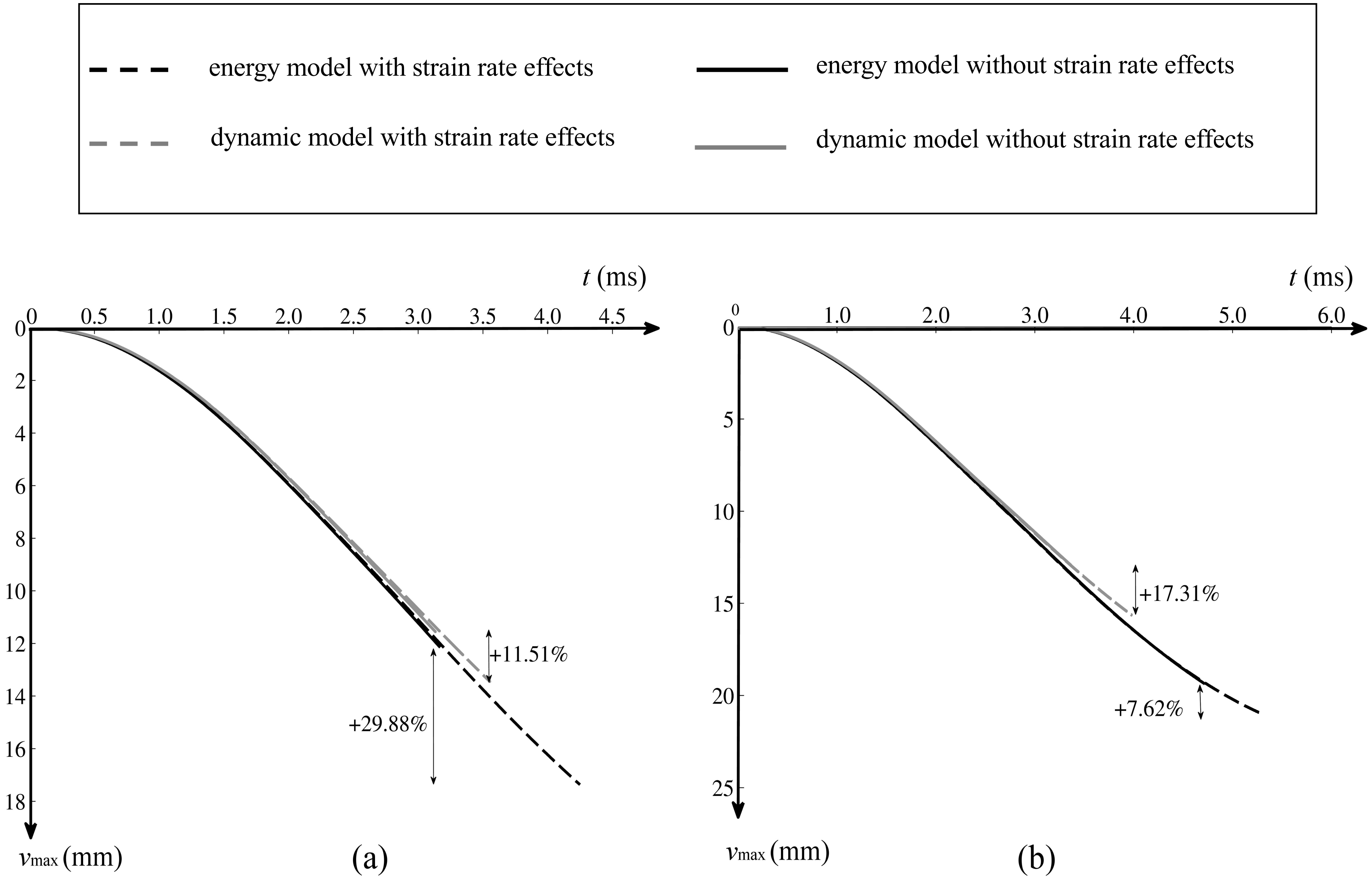
- What is the importance of Strain Rate Effects?





Khanna, Sanjeev K., and Ha TT Phan. "High Strain Rate Behavior of Graphene Reinforced Polyurethane Composites." Journal of Engineering Materials and Technology 137.2 (2015): 021005.

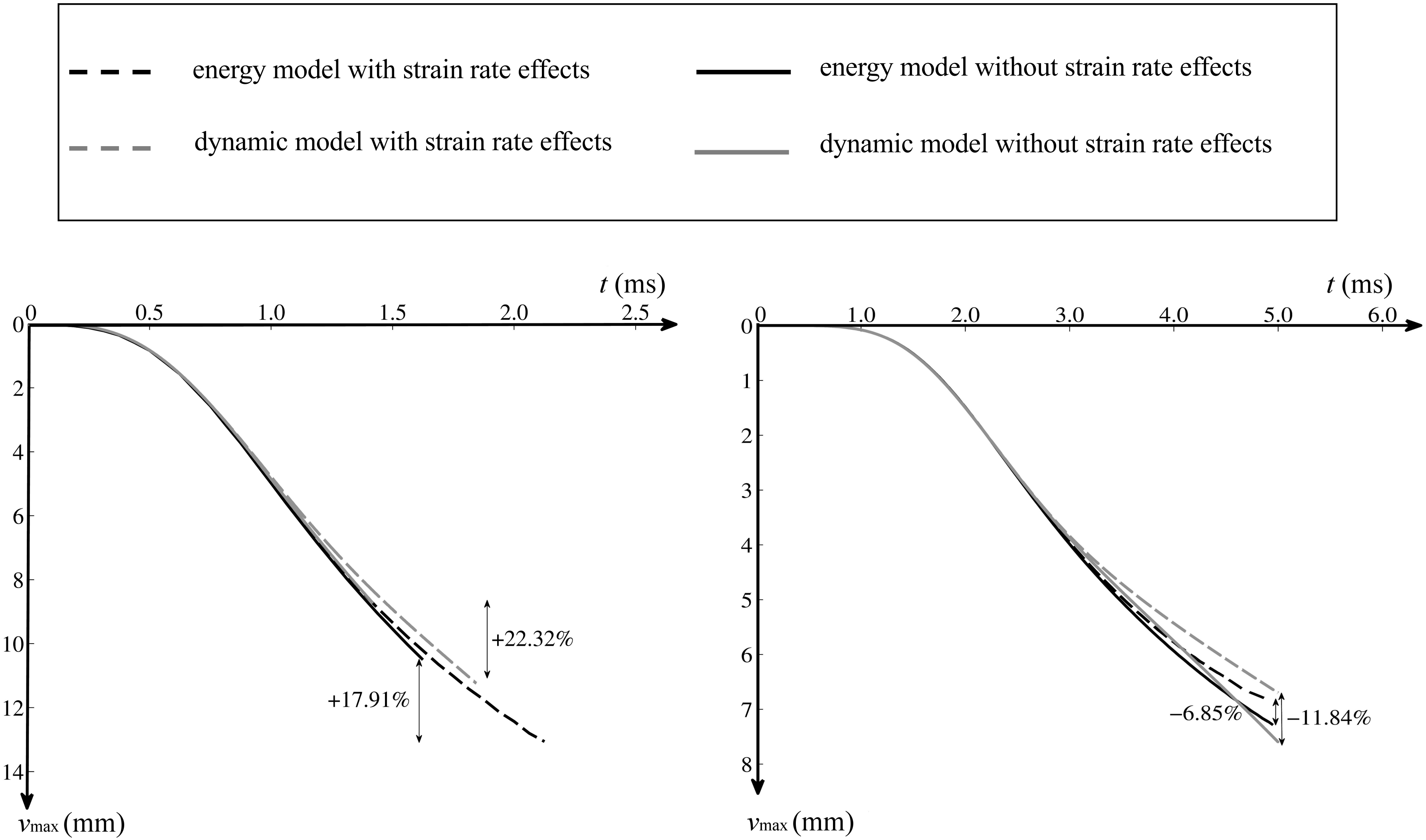
Case Study 1: Strain Rate Importance



B40-D5

B200-40/D3

Case Study 2: Strain Rate Importance



(c)
S1616 –h1.2

(d)
S1616 –h0.3

Outline

- **Main Aim of the Lecture**: *Simplified models for the flexural behaviour of R.C. beams subjected to explosion (and impulsive) verified with experimental results. In all models, account is taken of the effects of strain-rate.*

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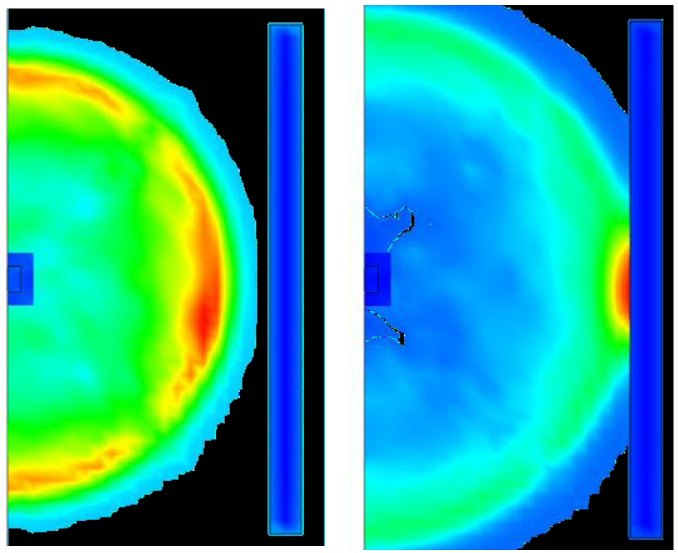
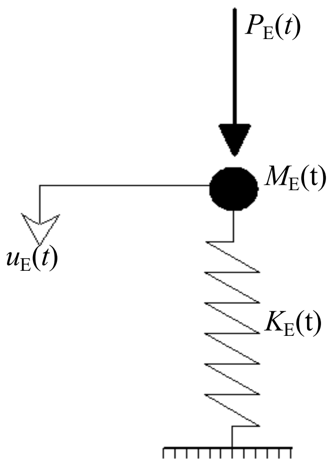
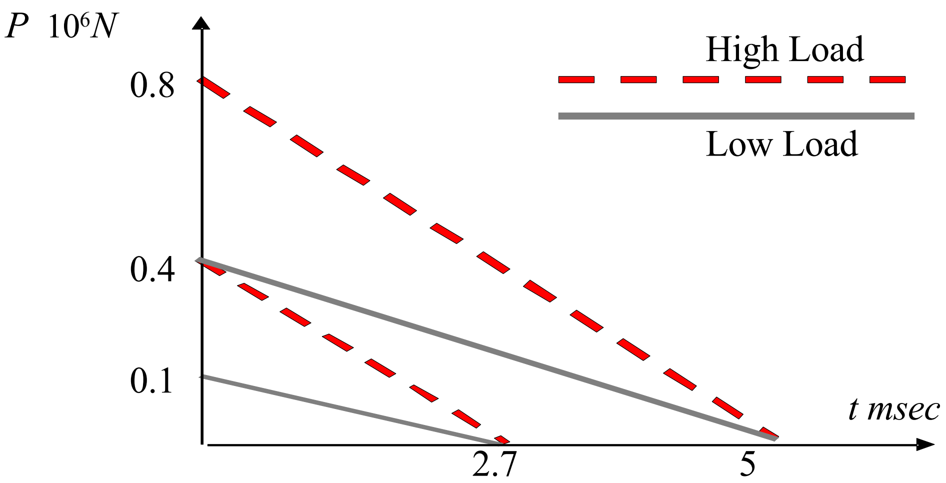
- Section 2: Energy Model
- Section 3: Sensitivity Analysis
- Section 4: Tower Building Case



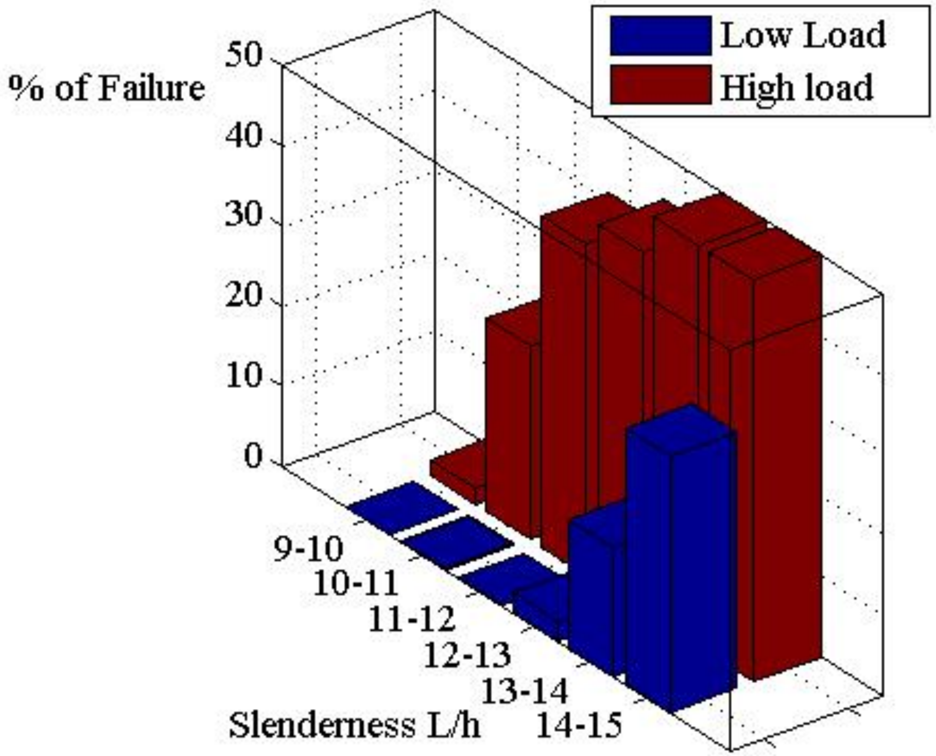
Sensitivity Analysis



Span Length	6 – 12 m
Slenderness h/L	1/9 – 1/15
Width	h/2.5
$\rho_s = A_s/bd$	0.005 – 0.01
$\rho_{As} = A_{ss}/A_s$	0.25 – 0.5
Concrete Strength	f_{cd} 20 – 40 MPa
Steel	B450 C



Sensitivity Analysis



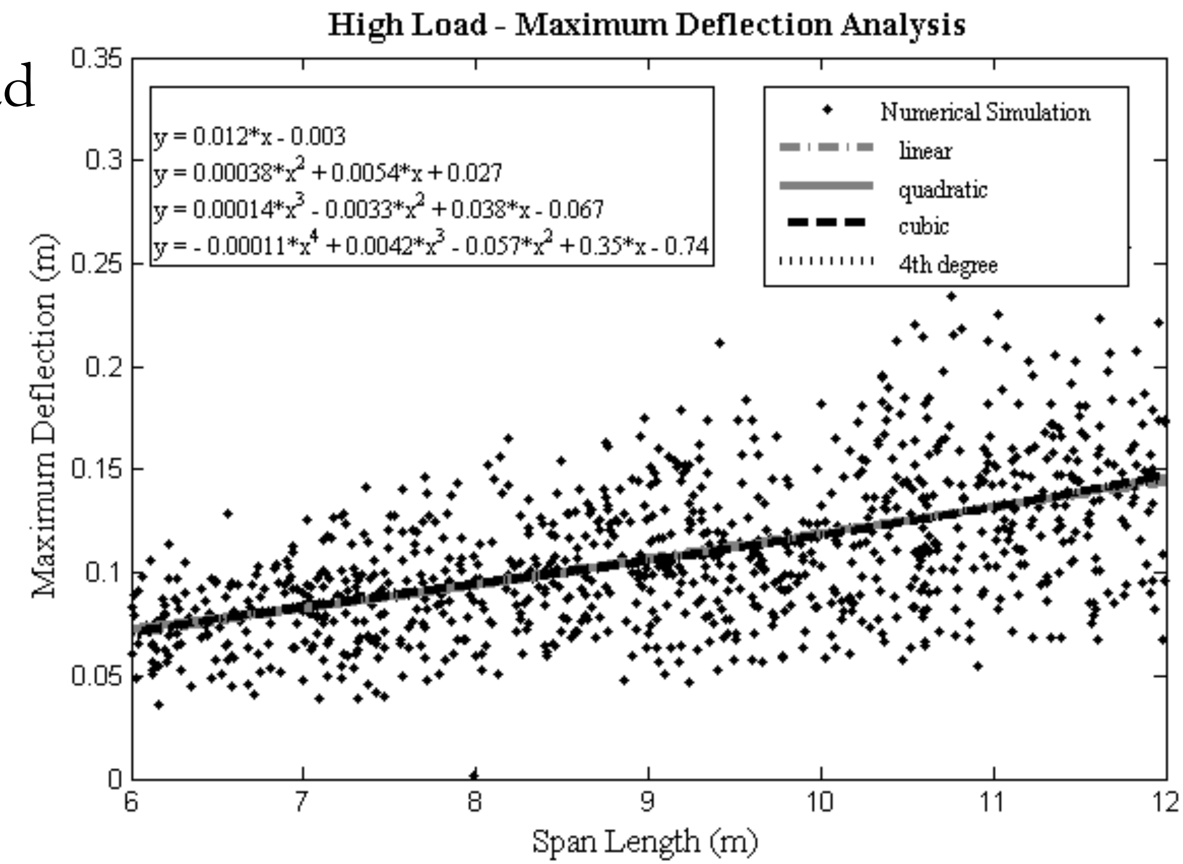
-4000 runs and some interesting results:

-50% of failure in case of High Load and slenderness greater than 12

-0% of failure in case of Low Load and slenderness lower than 13

Sensitivity Analysis

-High Load



Goodness of fit:

Function	**SSE m^2**	**R-square**	**Adjusted R-square:**	**RMSE m**
Linear	0.9585	0.2892	0.2884	0.03207
Quadratic	0.9576	0.2898	0.2883	0.03207
Cubic	0.9574	0.2900	0.2877	0.03208
4^{th} degree	0.9569	0.2903	0.2873	0.03209

Sensitivity Analysis

Fitting goodness:

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Sum of Squares Due to Error.

$$R_{-SQUARE} = \frac{SSR}{SST}$$

R-Square: ratio between the sum of squares regarding the mean of regression and the sum of squares regarding the mean of the response value.

$$adjusted\ R_{-SQUARE} = 1 - \frac{SSE\,(h-1)}{SST\,(v)}$$

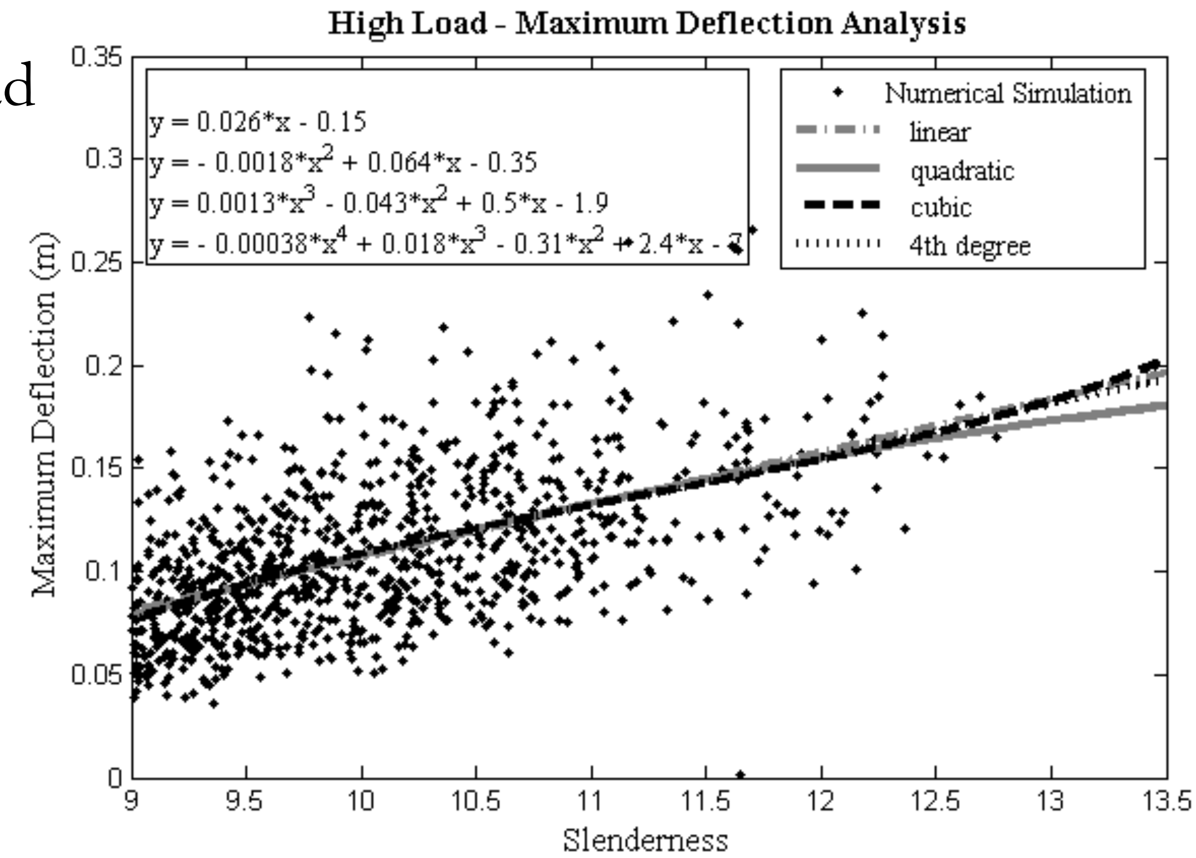
Adjusted R-square: it is an optimal indicator of fit validity when it is necessary to compare different models with different numbers of coefficients.

$$RMSE = \sqrt{\frac{SSE}{v}}$$

Root Mean Squared Error.

Sensitivity Analysis

-High Load

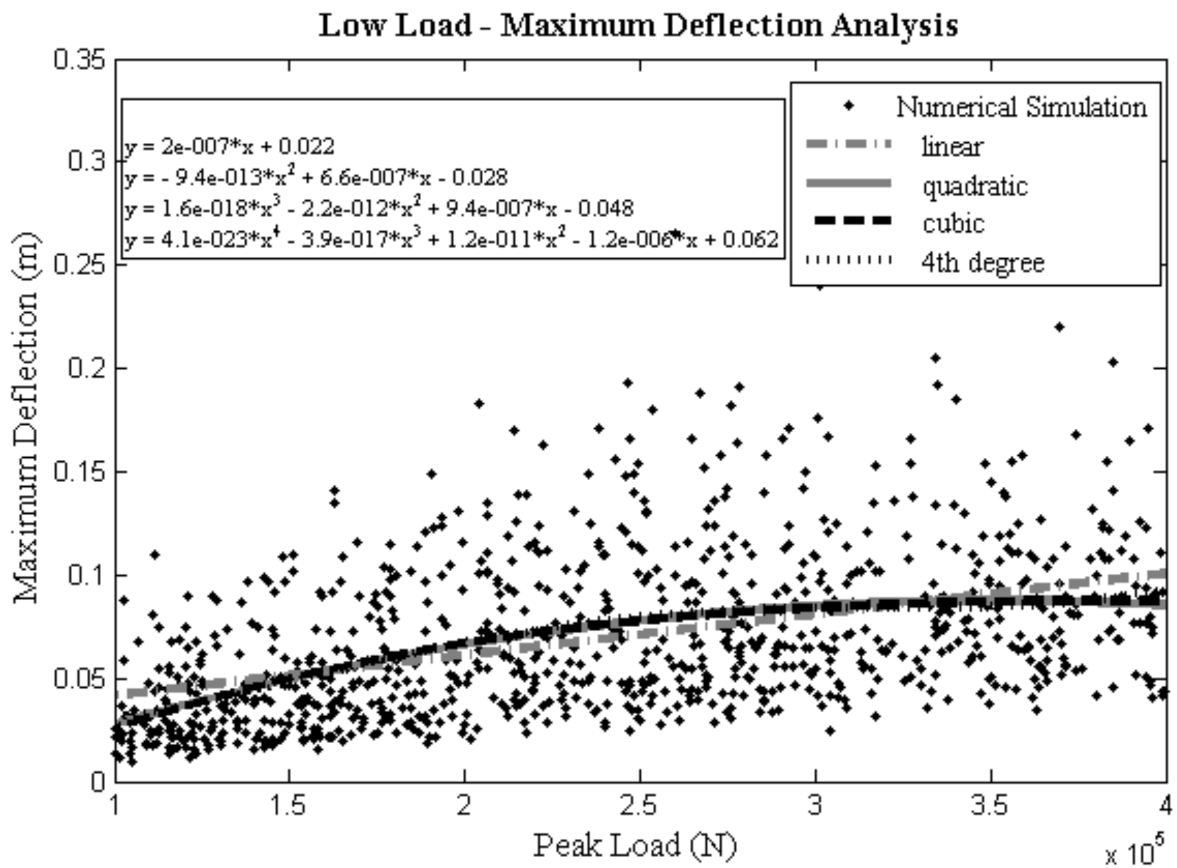


Goodness of fit:

Function	SSE m^2	R-square	Adjusted R-square:	RMSE m
Linear	0.9583	0.2893	0.2886	0.03207
quadratic	0.9562	0.2909	0.2893	0.03205
Cubic	0.9550	0.2918	0.2895	0.03204
4^{th} degree	0.9548	0.2919	0.2889	0.03206

Sensitivity Analysis

-Low Load

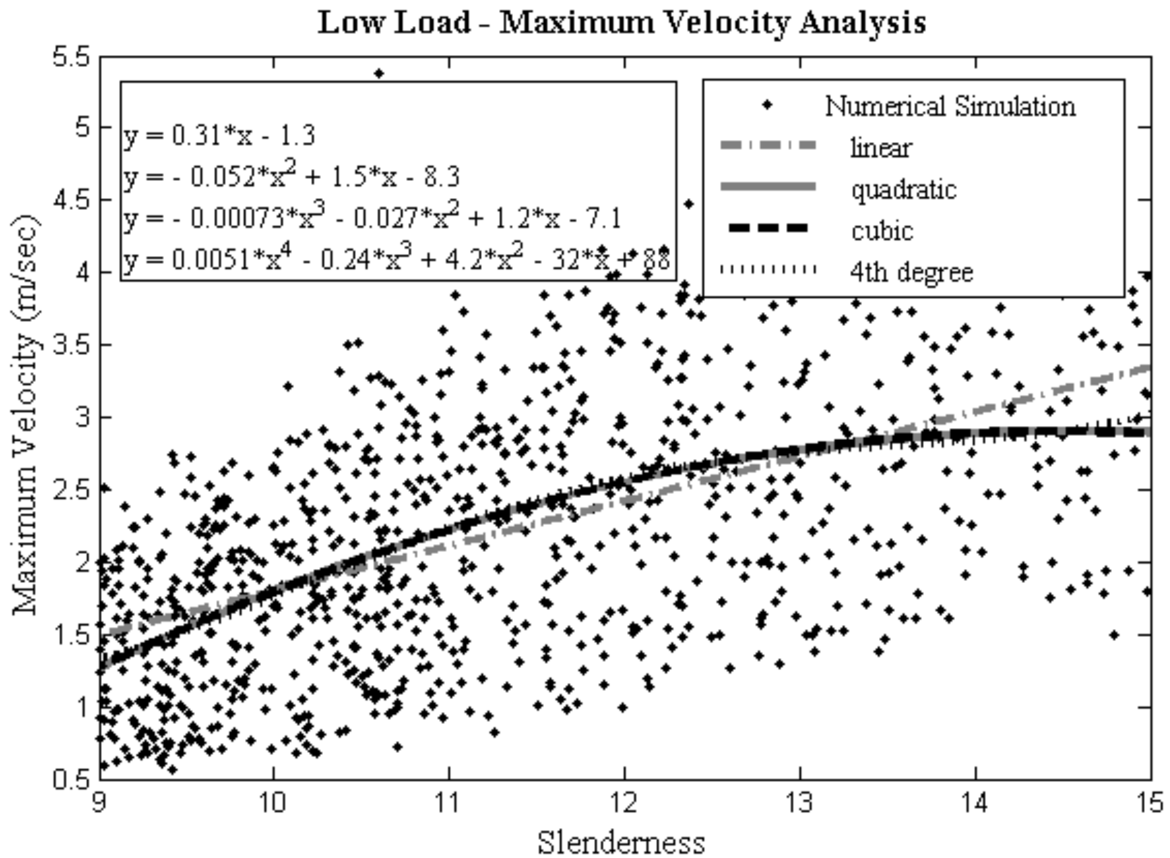


Goodness of fit:

Function	SSE m^2	R-square	Adjusted R-square:	RMSE m
Linear	2.358	0.1811	0.1807	0.03550
Quadratic	2.285	0.2064	0.2056	0.03496
Cubic	2.284	0.2069	0.2056	0.03496
4^{th} degree	2.279	0.2085	0.2068	0.03493

Sensitivity Analysis

-Low Load

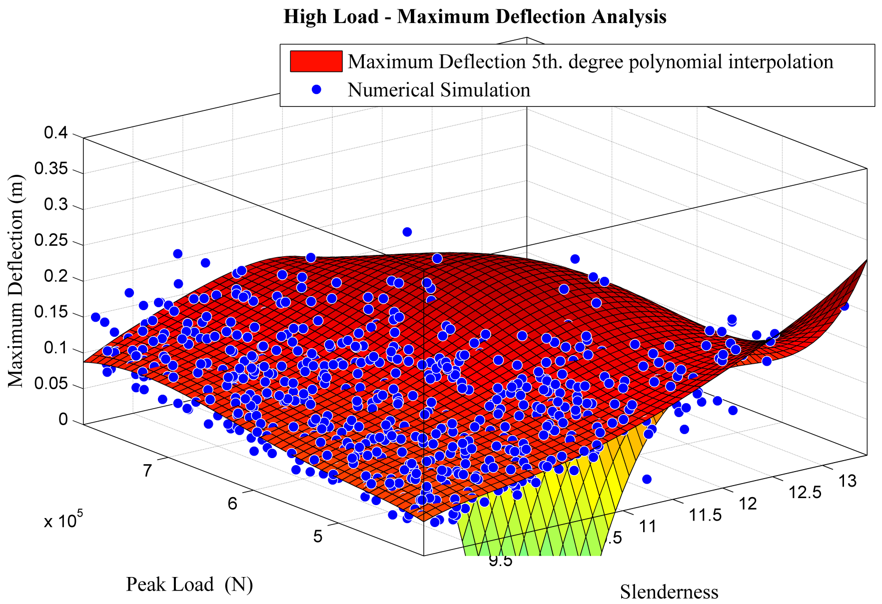


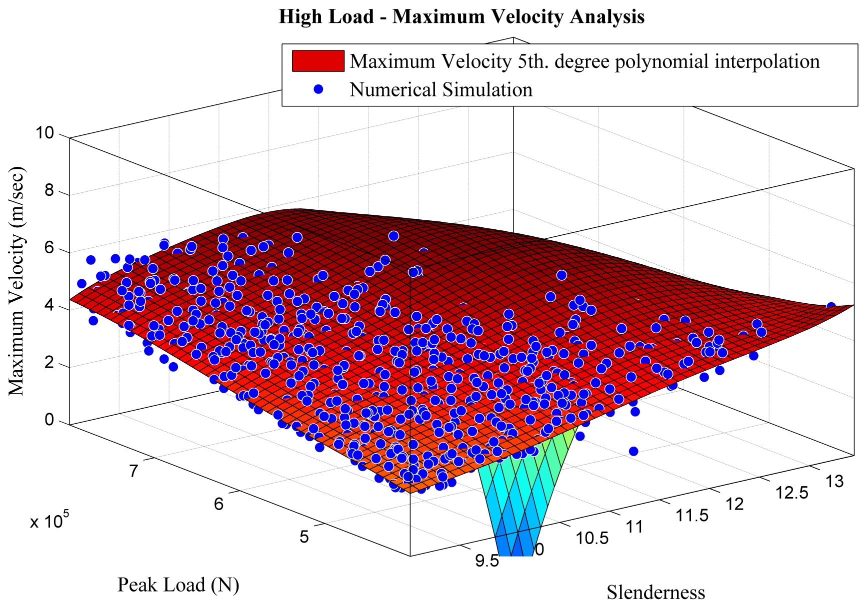
Goodness of fit:

Function	SSE m^2/sec^2	R-square	Adjusted R-square:	RMSE m/sec
Linear	1077	0.2919	0.2915	0.7587
Quadratic	1045	0.3132	0.3124	0.7474
Cubic	1045	0.3132	0.3121	0.7476
4^{th} degree	1043	0.3143	0.3128	0.7472

Sensitivity Analysis

-High Load





$$f(x,y) = p_0 + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2 + p_{30}x^3 + p_{21}x^2y + p_{12}xy^2 + p_{03}y^3 + p_{40}x^4 + p_{31}x^3y + $$
$$+ p_{22}x^2y^2 + p_{13}xy^3 + p_{04}y^4 + p_{50}x^5 + p_{41}x^4y + p_{32}x^3y^2 + p_{23}x^2y^3 + p_{14}xy^4 + p_{05}y^5$$

Deflection Goodness of fit:

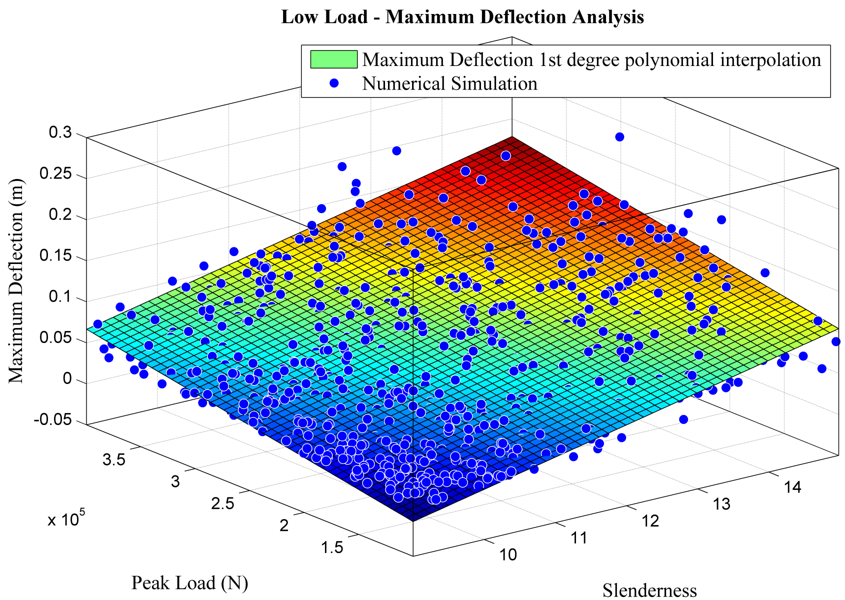
SSE m^2	R-square	AR-square:	RMSE m
0.6011	0.5503	0.5405	0.02567

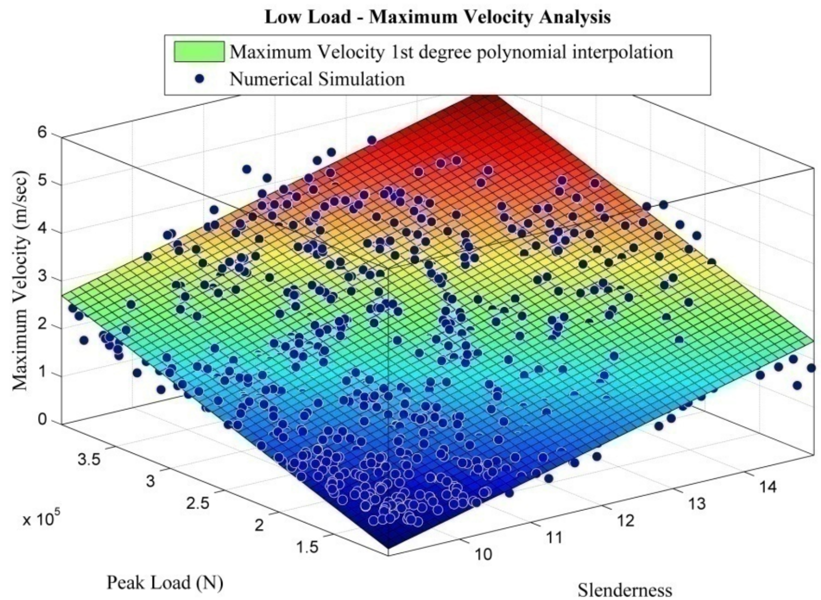
Velocity Goodness of fit:

SSE m^2/sec^2	R-square	A R-square:	RMSE m/sec
91.67	0.852	0.8487	0.317

Sensitivity Analysis

-Low Load:





$$f(x,y) = p_0 + p_{10}x + p_{01}y$$

Deflection Goodness of fit:

SSE m^2	R-square	A R-square:	RMSE m
0.7536	0.7383	0.738	0.02007

Velocity Goodness of fit:

SSE m^2/sec^2	R-square	A R-square:	RMSE m/se
132.1	0.9131	0.913	0.2658

Sensitivity Analysis

-High Load Max. Displacements:

x – y	Fit type	SSE m^2	R-SQUARE	AR-SQUARE	RMSE m	Coefficients
Span –Slend.	poly55	0.599829	0.551294	0.541454	0.025646	21
Slend- P.Load	poly55	0.601113	0.550334	0.540473	0.025673	21
Span-Slend.	poly44	0.604141	0.548069	0.541177	0.025654	15
Slend- P.Load	poly44	0.605854	0.546787	0.539876	0.025690	15
Span-Slend.	poly33	0.606089	0.546612	0.542191	0.025625	10
Span-Slend.	poly22	0.606666	0.546180	0.543733	0.025582	6
Slend- P.Load	poly33	0.60937	0.544157	0.539712	0.025694	10
Slend- P.Load	poly22	0.613641	0.540963	0.538487	0.025729	6
Span-Slend.	poly44	0.618171	0.541562	0.534578	0.025936	15
Slend- P.Load	poly11	0.629505	0.529095	0.528083	0.026017	3
Span-Slend.	poly11	0.633605	0.526028	0.525009	0.026102	3
Slend- R.Ratio	poly55	0.664026	0.507555	0.496768	0.026969	21
Span- P.Load	poly55	0.846643	0.372126	0.358372	0.030452	21
Slend.-C.Strength	poly55	0.938409	0.304072	0.288827	0.032060	21

Sensitivity Analysis

-High Load Max. Velocities:

x – y	Fit type	SSEm^2/sec^2	R-SQUARE	AR-SQUARE	RMSE m/sec	Coefficients
Slend.-P.Load	poly55	91.67459508	0.851965086	0.848718707	0.317049506	21
Slend.-P.Load	poly44	92.23189316	0.851065169	0.848793832	0.316970774	15
Slend.-P.Load	poly33	92.49950564	0.850633032	0.849176583	0.316569343	10
Slend.-P.Load	poly22	92.92596403	0.849944393	0.849135031	0.316612947	6
Slend.-P.Load	poly11	98.40803253	0.841092021	0.840750284	0.325292313	3
P.Load.-C.Stren.	poly33	357.6543056	0.422464597	0.416833157	0.622487849	10
P.Load – Span.	poly33	362.5423019	0.414571526	0.408863122	0.626727128	10
Slend.-Span	poly55	508.139387	0.179463294	0.161469068	0.746438464	21
Slend.-Span	poly44	510.7910993	0.175181345	0.162602411	0.745933857	15
Slend.-C.Stren.	poly33	511.2570454	0.174428941	0.166378952	0.74424993	10
Slend.-Span	poly33	512.3703241	0.172631233	0.164563716	0.745059803	10
Slend.-Span	poly22	515.5991811	0.167417318	0.162926581	0.745789462	6
Slend.-Span	poly11	529.3251071	0.145252876	0.14341471	0.75443143	3

Sensitivity Analysis

-Low Load Max. Displacements:

x – y	Fit type	SSE m^2	R-SQUARE	AR-SQUARE	RMSE m	Coefficients
Slend.-P.Load	poly55	0.58467087	0.796950663	0.794757906	0.017767865	21
Slend.-P.Load	poly44	0.591579941	0.794551224	0.793003171	0.017843657	15
Slend.-P.Load	poly33	0.59624080	0.792932562	0.791932236	0.017889756	10
Slend.-P.Load	poly22	0.601406477	0.791138583	0.790579233	0.017947828	6
Slend.-P.Load	poly11	0.75360261	0.738282651	0.738002739	0.020074761	3
Slend.-Span	poly55	1.249337712	0.566119663	0.561434130	0.02597284	21
Slend.-Span	poly44	1.255508783	0.563976523	0.560691093	0.025994832	15
Slend.-Span	poly33	1.264076134	0.561001182	0.558880415	0.026048348	10
Slend.-Span	poly22	1.265849822	0.560385201	0.559207872	0.026038678	6
Slend.-Span	poly11	1.327557523	0.53895484	0.538461744	0.026644395	3
P.Load-R.Ratio	poly11	2.128770294	0.260703041	0.259912349	0.033739885	3
C.Streng.-P.Load	poly11	2.351506343	0.183349423	0.182476000	0.035461106	3

Sensitivity Analysis

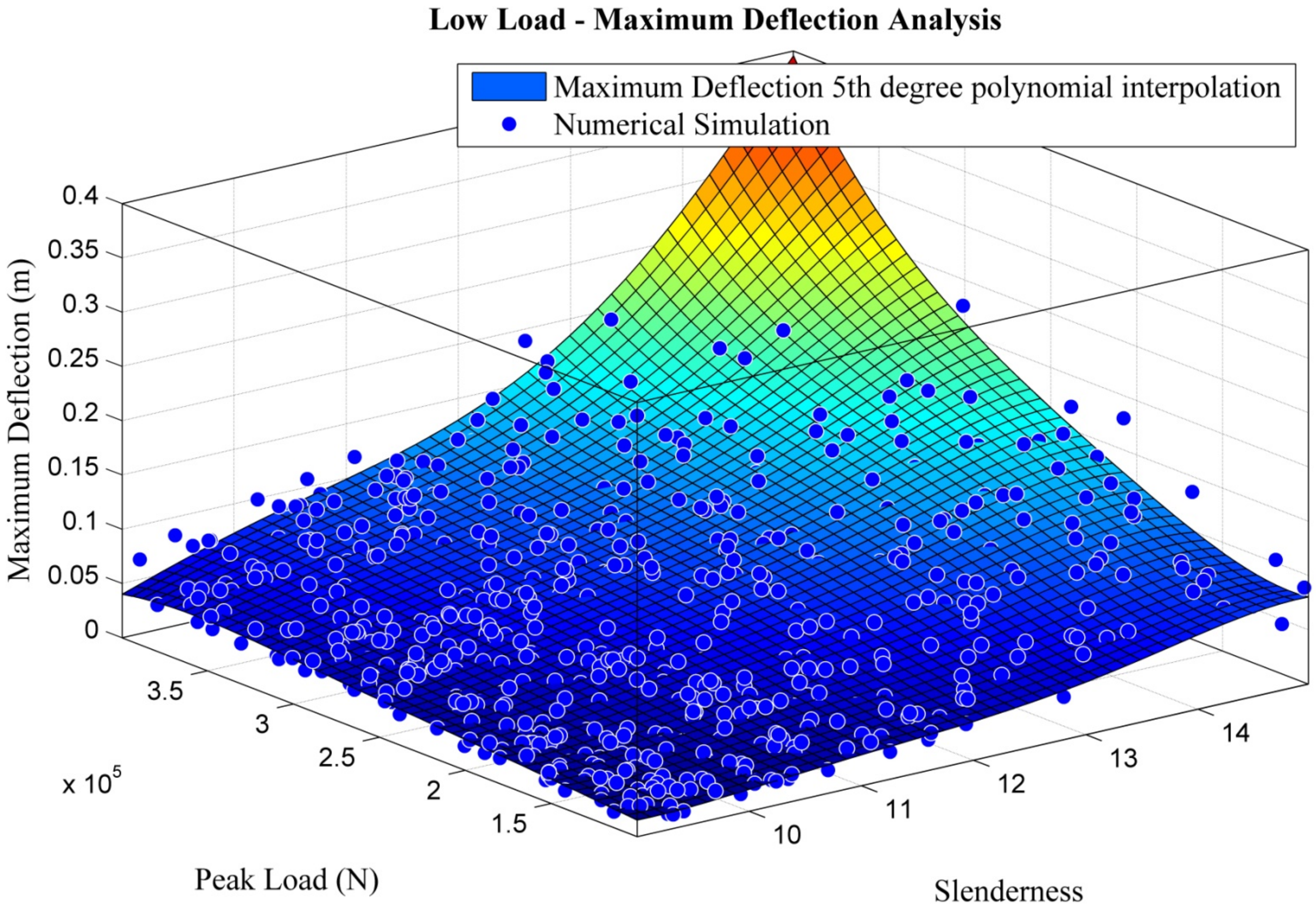
-Low Load Max. Velocities:

x – y	Fit type	SSE m^2/sec^2	R-SQUARE	AR-SQUARE	RMSE m/sec	Coefficients
Slend.-P.Load	poly55	74.15608153	0.951244314	0.950717795	0.200102675	21
Slend.-P.Load	poly44	74.91211988	0.950747238	0.95037612	0.200795136	15
Slend.-P.Load	poly33	75.21087461	0.950550815	0.95031193	0.200924961	10
Slend.-P.Load	poly22	75.41919317	0.950413851	0.950281055	0.200987377	6
Slend.-P.Load	poly11	132.1395093	0.913121725	0.913028807	0.265824828	3
Slend.-Span	poly55	1026.960884	0.324800050	0.317508474	0.744657315	21
Slend.-Span	poly44	1031.492396	0.321820699	0.316710629	0.745092446	15
Slend.-Span	poly33	1038.129148	0.317457208	0.314159900	0.746481871	10
Slend.-Span	poly22	1038.440420	0.317252555	0.315424094	0.745793567	6
R.Ratio-Slend.	poly11	1054.338630	0.306799897	0.306058507	0.750877786	3
Slend.-Span	poly11	1073.179019	0.294412834	0.293658195	0.757556944	3
C.Stren.-Slend.	poly11	1076.444277	0.292266012	0.291509077	0.758708542	3

Sensitivity Analysis

-Low Load- Best Fit

Goodness of fit:
SSE m^2
0.5847
R-square
0.797
Adjusted R-square:
0.7948
RMSE m
0.01777

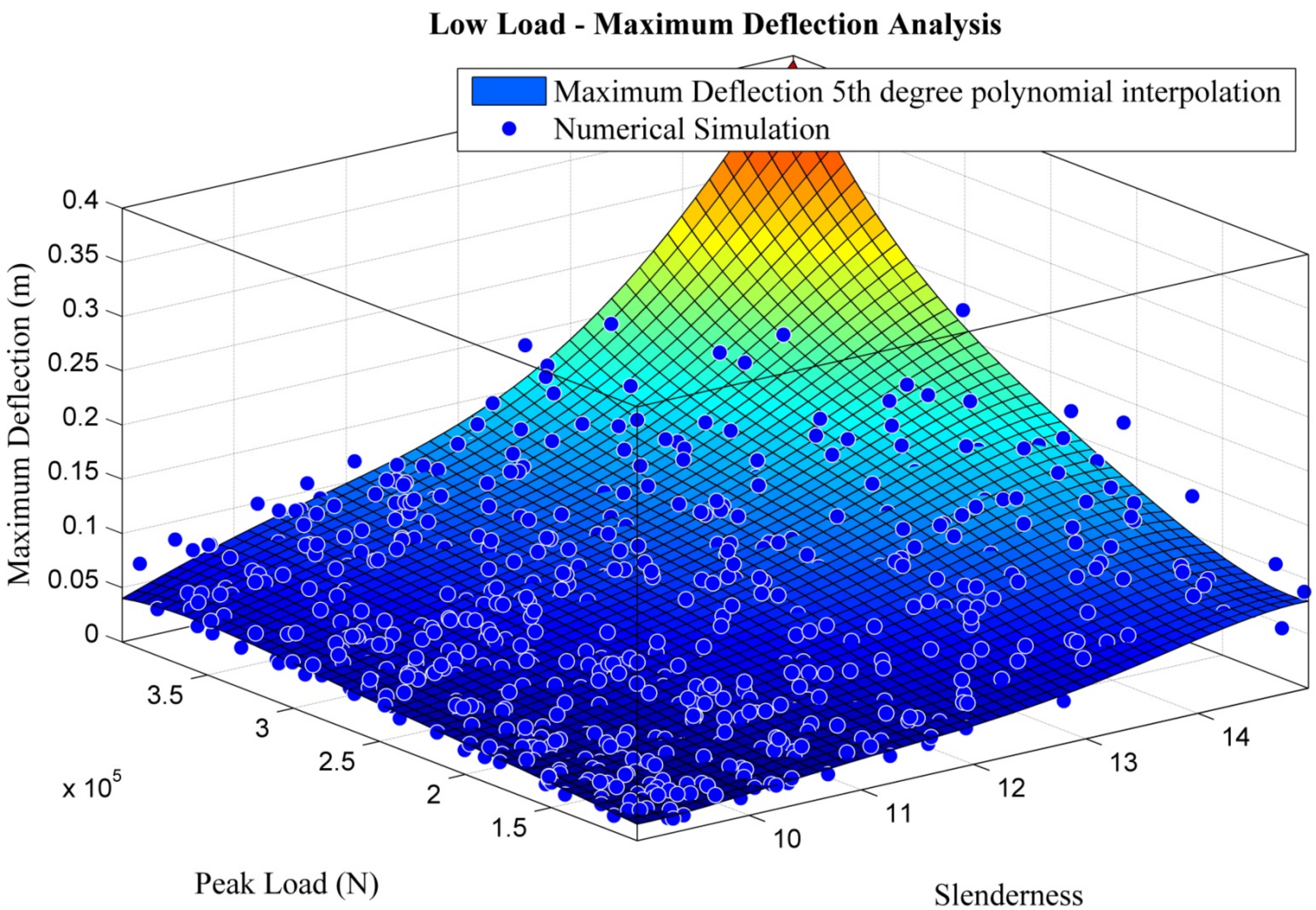


$$f(x,y) = p_0 + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2 + p_{30}x^3 + p_{21}x^2y + p_{12}xy^2 + p_{03}y^3 + p_{40}x^4 + p_{31}x^3y +$$
$$+ p_{22}x^2y^2 + p_{13}xy^3 + p_{04}y^4 + p_{50}x^5 + p_{41}x^4y + p_{32}x^3y^2 + p_{23}x^2y^3 + p_{14}xy^4 + p_{05}y^5$$

Sensitivity Analysis

-Low Load- Best Fit

Goodness of fit:
SSE m^2
0.5847
R-square
0.797
Adjusted R-square:
0.7948
RMSE m
0.01777

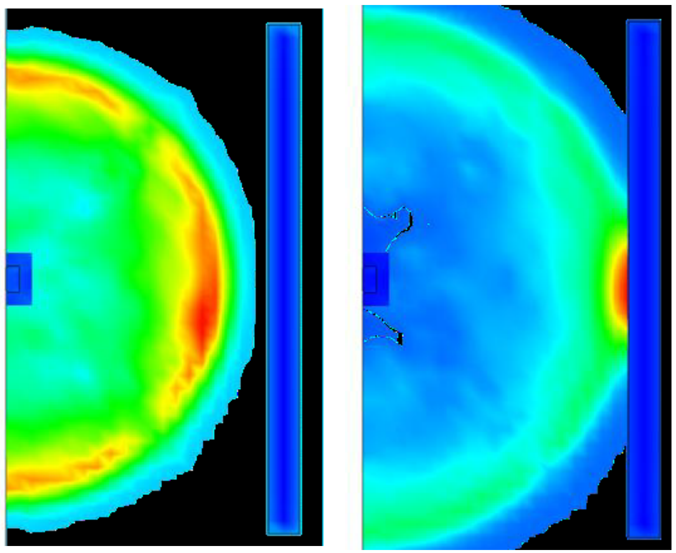


$$f(x,y) = p_0 + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2 + p_{30}x^3 + p_{21}x^2y + p_{12}xy^2 + p_{03}y^3 + p_{40}x^4 + p_{31}x^3y +$$
$$+ p_{22}x^2y^2 + p_{13}xy^3 + p_{04}y^4 + p_{50}x^5 + p_{41}x^4y + p_{32}x^3y^2 + p_{23}x^2y^3 + p_{14}xy^4 + p_{05}y^5$$

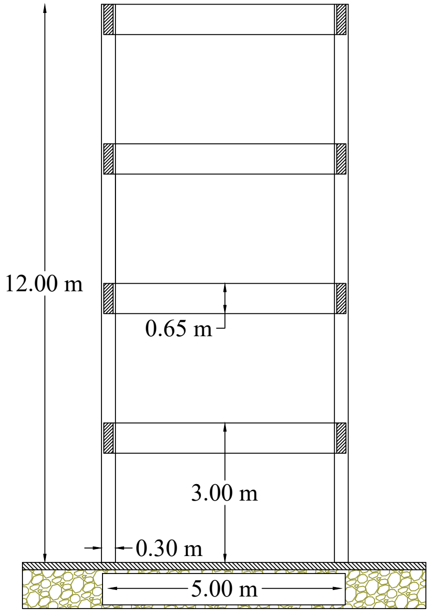
Outline

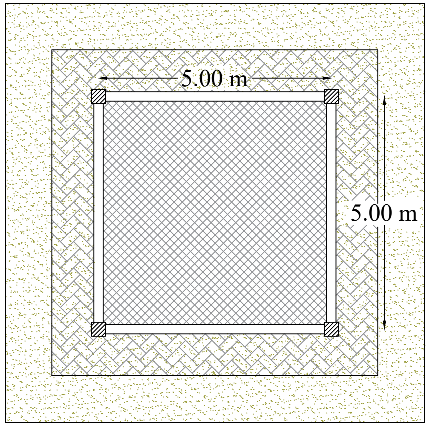
- **Main Aim of the Lecture**: *Simplified models for the flexural behaviour of R.C. beams subjected to explosion (and impulsive) verified with experimental results. In all models, account is taken of the effects of strain-rate.*

- Section 1: Dynamic Models
- Section 2: Energy Model
- Section 3: Sensitivity Analysis
- Section 4: Tower Building Case



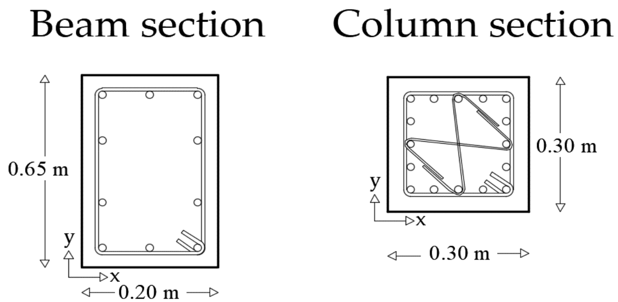
Stochino F., Attoli A., Concu G., "Fragility curves for RC structure under blast load considering the influence of seismic demand", Applied Sciences, 10, article number 445, (2020).

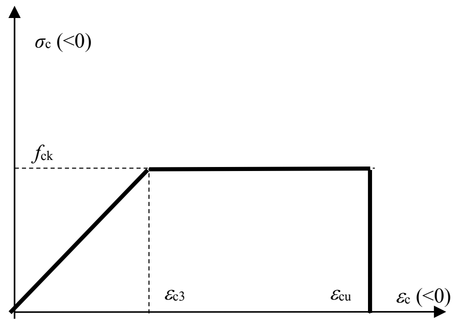


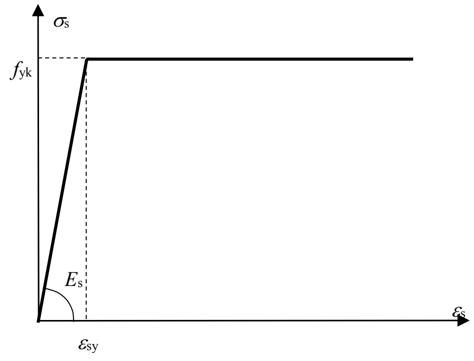


A framed RC structure with squared cross section has been considered as a case study.

This kind of structure can serve as watchtower in a military environment.



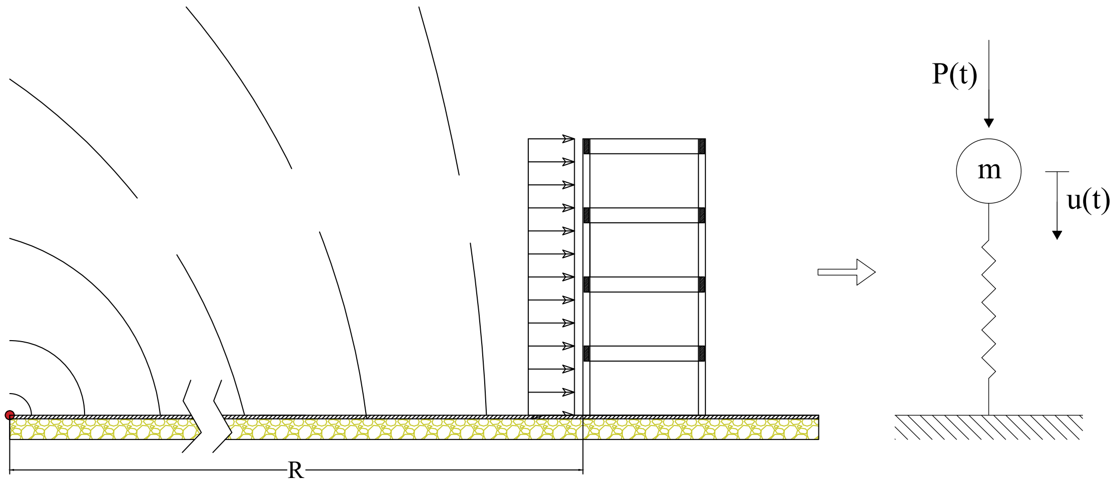




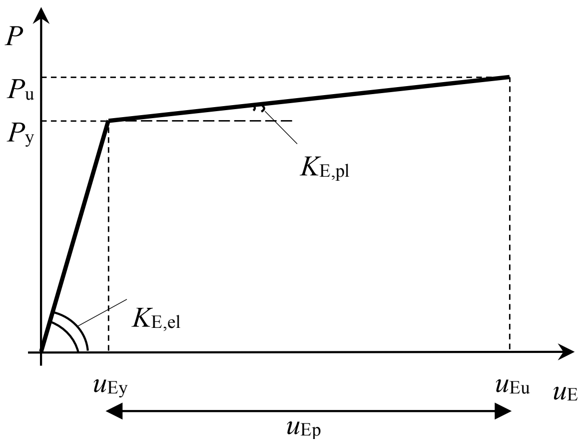
f_{ck} (MPa)	e_{c3} ‰	e_{cu}‰	f_{yd} (MPa)	e_{sy} ‰
28	1.75	3.5	450	2.9

Fragility analysis of a RC Structure 2/7

Hemispheric Explosion above ground



Bilinear SDOF model



SDOF equations of motion

$$\mathrm{M}_{E,el}\frac{d^2v_E(t)}{dt^2} + K_{E,el}u_E(t) = P_E(t) \quad for\ 0 \le u_E \le u_{Ey}$$

$$\mathrm{M}_{E,pl}\frac{d^2v_E(t)}{dt^2} + K_{E,pl}u_E(t) + [K_{E,el} - K_{E,pl}]u_{Ey} = P_E(t) \quad for\ u_{Ey} < u_E \le u_{Eu}$$

Scaled distance as intensity measure

$$Z = \frac{R}{W^{\frac{1}{3}}}$$

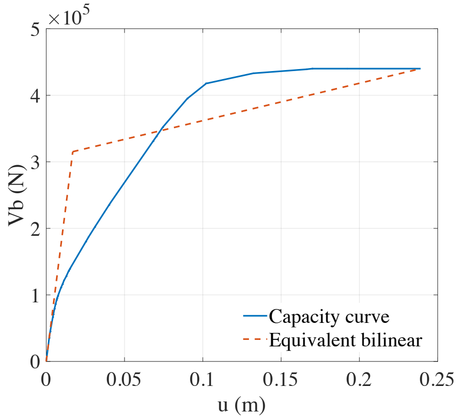
distance

explosive charge

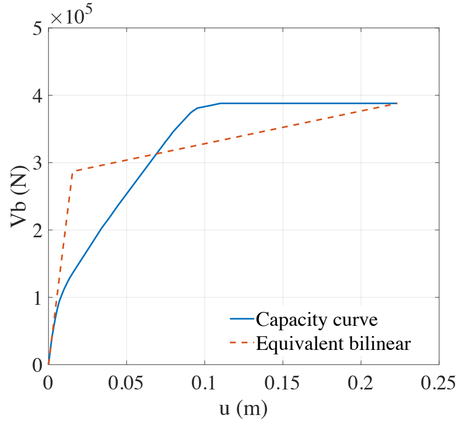
Fragility analysis of a RC Structure 3/7

Pushover analysis yields to capacity curve.

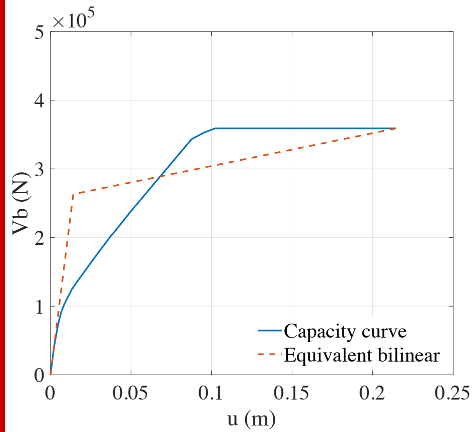
Bilinear SDOF model.



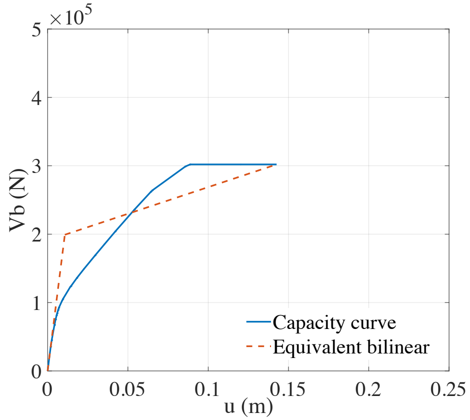
L'Aquila



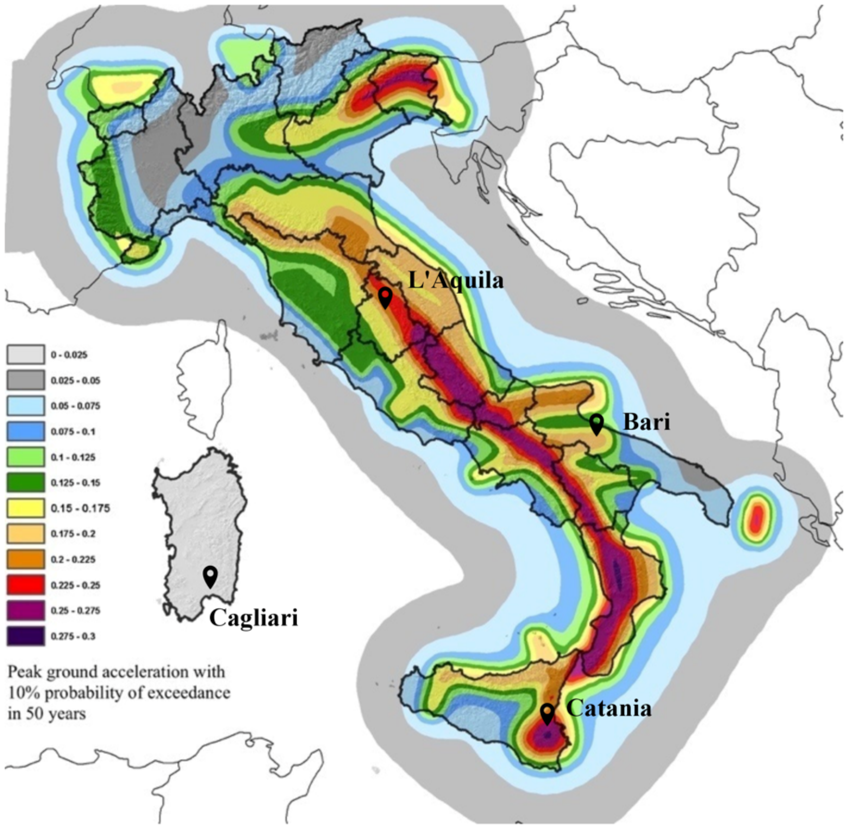
Catania



Bari



Cagliari

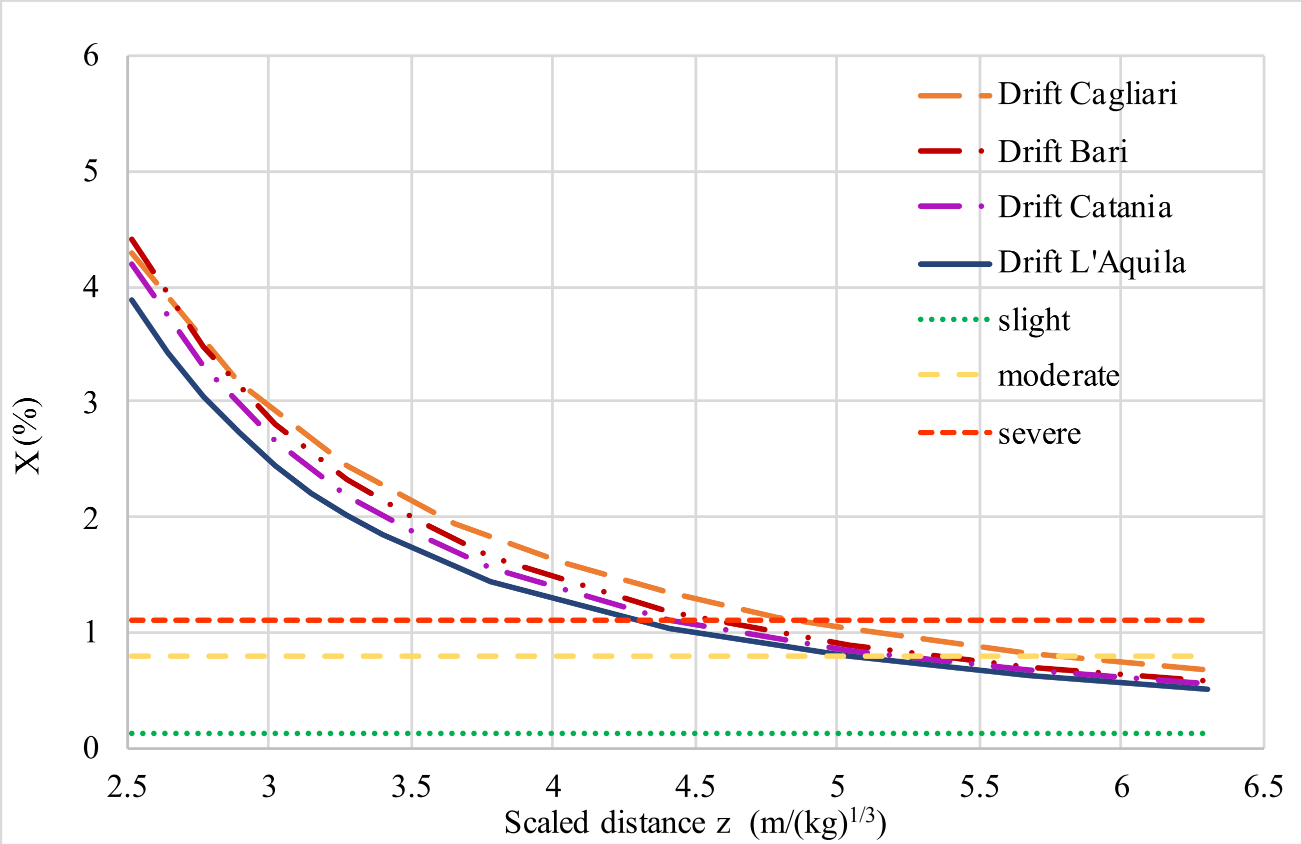


Fragility analysis of a RC Structure 4/7

Maximum drift damage thresholds* $X = \frac{u_{MAX}}{h}$

Slight Damage	Moderate Damage	Severe Damage
0.0012	0.0080	0.011

Maximum drift for the structure under blast load - 500 kg TNT

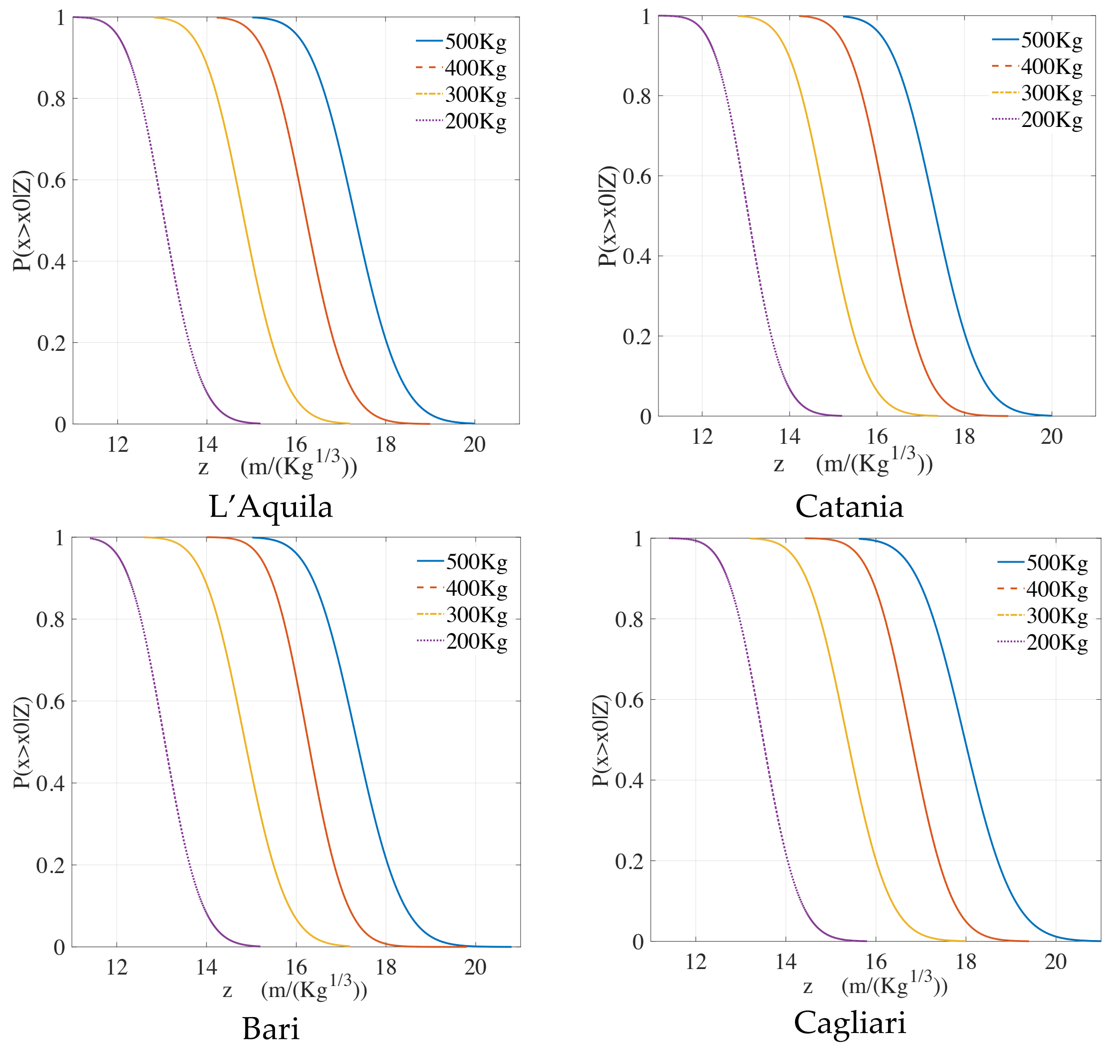


* Akkar, S., Sucuoğlu, H., Yakut, A. Displacement-based fragility functions for low-and mid-rise ordinary concrete buildings. EarthqSpect 2005 21(4), 901-927.

Fragility analysis of a RC Structure 5/7

Description	COV	Distribution
Stand-off distance	0.05	Lognormal
Explosive mass	0.15	Lognormal

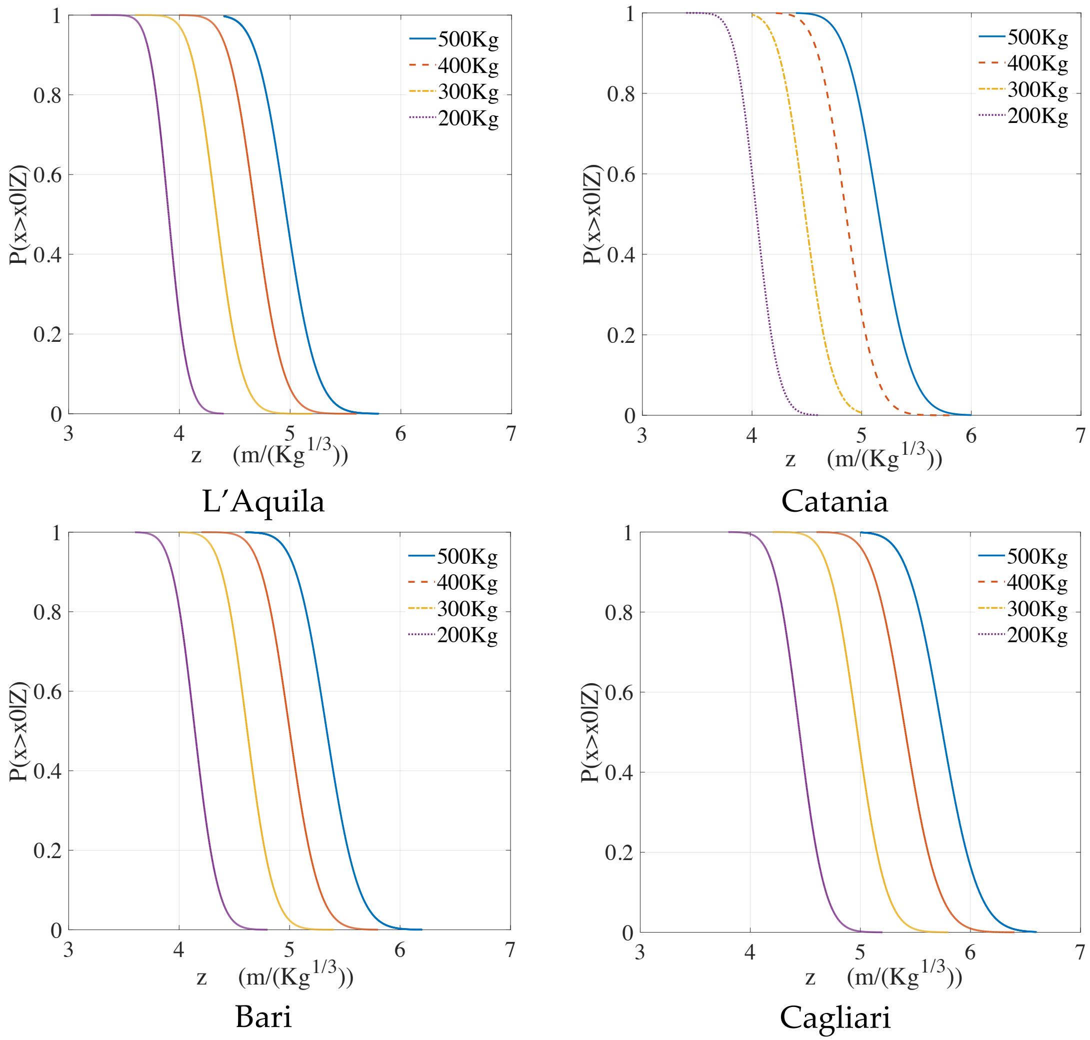
Slight Damage	Moderate Damage	Severe Damage
0.0012	0.0080	0.011



Fragility analysis of a RC Structure 6/7

Description	COV	Distribution
Stand-off distance	0.05	Lognormal
Explosive mass	0.15	Lognormal

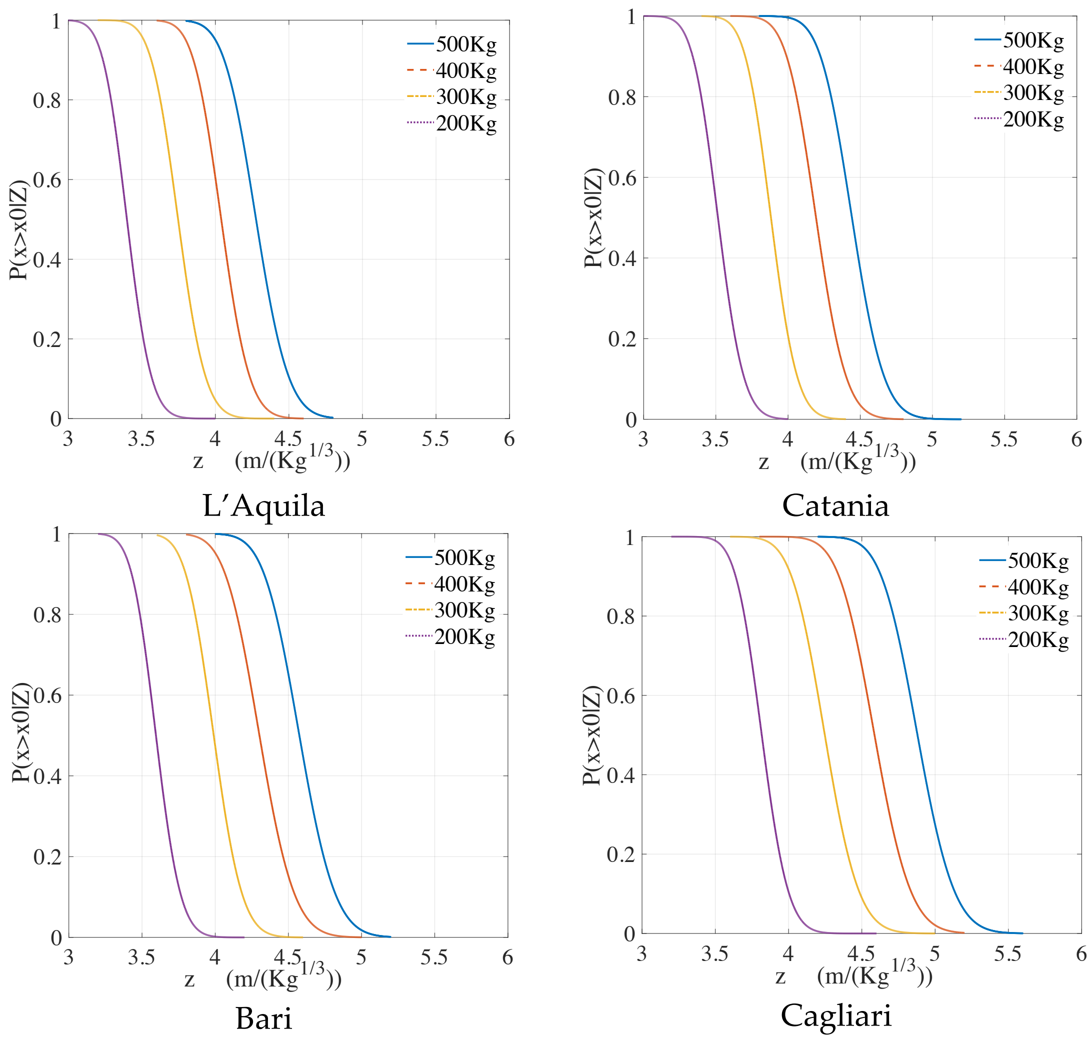
Slight Damage	Moderate Damage	Severe Damage
0.0012	0.0080	0.011



Fragility analysis of a RC Structure 7/7

Description	COV	Distribution
Stand-off distance	0.05	Lognormal
Explosive mass	0.15	Lognormal

Slight Damage	Moderate Damage	Severe Damage
0.0012	0.0080	0.011



Literature References

•Stochino F., Attoli A., Concu G., ”Fragility curves for RC structure under blast load considering the influence of seismic demand”, Applied Sciences, 10, article number 445, (2020).

•Carta, G., and F. Stochino. "Theoretical models to predict the flexural failure of reinforced concrete beams under blast loads." Engineering structures 49 (2013): 306-315.

•Stochino, F., and G. Carta. "SDOF models for reinforced concrete beams under impulsive loads accounting for strain rate effects." Nuclear Engineering and Design 276 (2014): 74-86.

•Stochino, Flavio. "RC beams under blast load: Reliability and sensitivity analysis." Engineering Failure Analysis 66 (2016): 544-565.

•Stochino, Flavio. "Flexural models of reinforced concrete beams under blast load." (2013), PhD Thesis.

Conclusions

•The ***smooth non linear relationship*** between bending moment and curvature yield a nonlinear equation of motion quite easy to integrate. Continuous beam model is capable of accurate and wide results concerning the displacements and curvature as shown by comparison with experimental data.

•Taking into account Strain Rate effects requires a greater computational effort, but it is of paramount relevance to model the mechanical behaviour of structures under blast load.

•***SDOF model*** is more convenient than the continuous beam model from a computational point of view, but it is less accurate.

•***Energy Model*** produces excellent results for what concerns midspan deflection. It can be improved adding more terms to the series representing the deformed shape.

•The sensitivity analysis have shown that the most significant parameters in the response are the ***slenderness***, and the ***peak load magnitude***. It 's interesting how simple 1st degree polynomial have obtained low Root mean square (RMSE = 0.02), confirming the significance of the parameters considered in the analysis.

•Probabilistic Developments.